Theoretical Models for Discount for Lack of Marketability (DLOM)

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The common shares of a closely held private company are distinguished from the common shares of a publicly held company by their lack of marketability. Common shares of privately held companies are not traded on organized exchanges; therefore, a seller of such shares may incur significant costs and or delays in realizing the cash proceeds from the sale of the shares. In the valuation of closely held private companies, a discount (either an amount or percentage) is deducted from the nominal value of the common shares to reflect this lack of marketability.

This discount for lack of marketability (DLOM) may be estimated either by the examination of empirical models based on actual transactions, or theoretical models. We shall consider a class of theoretical models here. These models are based on option pricing theory, applied according to various assumptions.

The Chaffee Model

This model,[1] also known as the "European Put Option Model," estimates the DLOM as the value of a European-style put option on the common shares with a strike price equal to the share price on the valuation date. The Black-Scholes-Merton model is used to calculate the put option price. In order to apply the model, the following inputs are required:

1. Share price
2. Strike price
3. Expected option life
4. Risk-free interest rate
5. Dividend yield (if any)
6. Expected volatility of share price

In practice, the strike price is set equal to the share price on the valuation date, while the expected option life or term is the estimated holding period or delay incurred in selling the shares. Dividend yield will reflect any dividends paid by the actual shares under consideration. The risk-free interest rate over the option term may be estimated from the yield of US Treasury of other Sovereign debt issues. Determination of expected volatility is more problematic. Since the subject stock does not actively trade, the expected volatility must be estimated from the data of "comparable" publicly traded companies.

Here is an example of the Chaffee European Put Option Model for DLOM:

Share price	$30.00
Strike price	$30.00
Expected life	1.00 years
Dividend Yield	0.00%
Risk-Free Yate	2.00%
Expected Volatility	50.0%
Value of Put Option	$5.57
DLOM	18.57%

The Longstaff Look-Back Put Option Model

This model[2] provides an estimate of the DLOM assuming the seller of the shares has perfect market knowledge, and is able to time his sale to maximize the proceeds. The so-called "look-back put option" allows the seller to exercise his put at the end of the holding period at the highest or best price attained by the stock in the period from valuation date to expiration. This allows the seller to obtain a better price, but the option itself is more costly.

Using the example conditions presented above, we use FinTools™ Exotics XL to calculate the value of the look-back put.

Value of Put Option	$13.54
Longstaff DLOM	45.12%

The Finnerty Average-Strike Put Option Model

The Finnerty model[3] assumes that the put option is struck at the average price of the stock over the period from valuation date to expiration date. The seller is not assumed to have any special market timing ability. We use FinTools Exotics XL to calculate the value of a put struck at the projected arithmetic mean of the share price over the life of the option, with exercise taking place on the date of expiration.

Value of Put Option	$3.29
Finnerty DLOM	10.97%

Comparison of the Models

It is instructive to make a comparison of the different models over a range of holding periods and volatilities. In common practice, the Finnerty model is generally accepted, although there are recognized shortcomings related to the estimate of volatility used.

DLOM from Chaffee European Put Model

Chaffee	20%	30%	40%	50%	60%	70%	80%
0.25	3.73%	5.72%	7.70%	9.68%	11.65%	13.61%	15.56%
0.50	5.13%	7.92%	10.70%	13.47%	16.22%	18.96%	21.67%
0.75	6.13%	9.53%	12.92%	16.28%	19.61%	22.90%	26.16%
1.00	6.94%	10.84%	14.72%	18.57%	22.37%	26.12%	29.80%
2.00	9.17%	14.58%	19.93%	25.17%	30.29%	35.26%	40.06%
3.00	10.64%	17.12%	23.49%	29.69%	35.66%	41.37%	46.80%
4.00	11.70%	19.03%	26.19%	33.08%	39.65%	45.85%	51.63%
5.00	12.51%	20.53%	28.32%	35.76%	42.75%	49.25%	55.21%

DLOM from Longstaff Look-Back Put Model

	Longstaff	20%	30%	40%	50%	60%	70%	80%
	0.25	7.97%	12.26%	16.69%	21.26%	25.96%	30.81%	35.79%
	0.50	11.25%	17.50%	24.03%	30.83%	37.92%	45.30%	52.97%
	0.75	13.75%	21.58%	29.81%	38.47%	47.57%	57.12%	67.13%
	1.00	15.85%	25.04%	34.79%	45.12%	56.04%	67.57%	79.73%
	2.00	22.27%	35.93%	50.75%	66.78%	84.05%	102.63%	122.56%
	3.00	27.10%	44.42%	63.53%	84.49%	107.40%	132.34%	159.41%
	4.00	31.08%	51.64%	74.61%	100.11%	128.27%	159.23%	193.10%
	5.00	34.53%	58.03%	84.58%	114.36%	147.52%	184.26%	224.72%

DLOM from Finnerty Average-Strike Put Model

Finnerty	20%	30%	40%	50%	60%	70%	80%
0.25	2.17%	3.32%	4.47%	5.63%	6.78%	7.94%	9.10%
0.50	3.00%	4.62%	6.24%	7.87%	9.51%	11.16%	12.82%
0.75	3.60%	5.58%	7.57%	9.57%	11.58%	13.60%	15.65%
1.00	4.08%	6.36%	8.65%	10.97%	13.30%	15.65%	18.03%
2.00	5.44%	8.64%	11.87%	15.15%	18.47%	21.86%	25.31%
3.00	6.36%	10.23%	14.17%	18.19%	22.30%	26.51%	30.81%
4.00	7.04%	11.47%	16.00%	20.65%	25.42%	30.32%	35.35%
5.00	7.57%	12.47%	17.52%	22.71%	28.07%	33.58%	39.20%

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February 2015

[1] Chaffee, "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations," ***Business Valuation Review***, December 1993, pp. 182-6

[2] Longstaff, "How Much Can Marketability Affect Security Values?" ***The Journal of Finance***, December 1995, pp. 1767-74

[3] Finnerty, "An Average-Strike Put Option Model for the Marketability Discount," ***The Journal of Derivatives***, Summer 2012, pp. 53-70