VALUE AT RISK FOR LONG/SHORT POSITIONS

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The objective of this paper is to provide a methodology for the computation of the value at risk (VaR) of a portfolio containing both long and short positions. The first sections present the definition of VaR and the usual approximations used for VaR estimates. The following sections provide VaR estimates for portfolio having only long (or short) positions, and VaR estimates for portfolio having both long and short positions.

VALUE AT RISK DEFINITION

According to Jorion (1997; page 87), for a given (1-α)% confidence level (e.g., 95%) the definition of VaR relative to the mean is:

$$VaR = E[Portfolio] - q_\alpha \qquad (1)$$

where:

E[Portfolio]	=	expected value of the portfolio on the date of interest
q_α	=	the α quantile (e.g., 5%) of the portfolio value on the date of interest

The time horizon (i.e., the time interval between today and the future date of interest) and the confidence level (e.g., 95%) are the two parameters used in the definition of VaR.

Based on the definition of VaR it results that, with (1-α)% confidence level (e.g., 95%), the losses for the selected time horizon will not exceed the value of VaR. This is the reason why VaR is commonly related to a "confidence level." However, it is more adequate to refer to VaR as a quantile estimate. Because VaR is an estimate, it is possible and useful to provide a "confidence interval" for VaR. This "confidence interval" built around the estimated VaR has its own "confidence level." Labeling VaR as a quantile removes the confusion of a "confidence interval" built around a "confidence level." It is less confusing to deal with a "confidence interval" built for a quantile. However, the usage of "confidence level" as a parameter for VaR is widely accepted by the industry and academia.

APPROXIMATIONS USED FOR VaR ESTIMATES

Longerstaey and Spencer (1996) define the continuously compounded returns ($r_{i,j}$) and the percent returns ($R_{i,j}$) as follows:

$$r_{i,j} = \ln\left(\frac{S_{i,j}}{S_{i-1,j}}\right) \qquad (2)$$

$$R_{i,j} = \left(\frac{S_{i,j} - S_{i-1,j}}{S_{i-1,j}}\right) \qquad (3)$$

where $\mathbf{S_{i,j}}$ is the value of asset **j** at time $\mathbf{t_i}$. It should be noted that the return rates are equal to the corresponding returns divided by the time interval $\mathbf{\Delta t_i = t_i - t_{i-1}}$.

The correct temporal aggregation of these returns is as follows (Longerstaey and Spencer 1996):

$$r_j = \sum_{i=1}^{N} r_{i,j} \qquad (4)$$

$$R_j = \prod_{i=1}^{N} (1 + R_{i,j}) - 1 \qquad (5)$$

Similarly, the correct cross-section aggregation is as follows (Longerstaey and Spencer 1996):

$$r_i = \ln\left(\sum_{j=1}^{N} w_j \cdot e^{r_{i,j}}\right) \qquad (6)$$

$$R_i = \sum_{j=1}^{N} w_j \cdot R_{i,j} \qquad (7)$$

where $\mathbf{w_j}$ is the fraction of asset **j** with respect to the total portfolio.

The continuously compounded returns are used in RiskMetrics as the basis for all computations (Longerstaey and Spencer 1996). In practice, RiskMetrics assumes that a portfolio return is a weighted average of continuously compounded returns (Longerstaey and Spencer 1996):

$$r_i = \sum_{j=1}^{N} w_j \cdot r_{i,j} \qquad (8)$$

It should be noted that this weighted average of continuously compounded returns - used as an approximation by RiskMetrics (Longerstaey and Spencer 1996) - is different with respect to the correct cross-section aggregation mentioned on the same page 49 of Longerstaey and Spencer (1996).

Another approximation largely used for VaR computations is (Longerstaey and Spencer 1996; page 8):

$$e^{x} \cong 1 + x \qquad (9)$$

For the purpose of VaR computations, some authors (Longerstaey and Spencer 1996; Jorion 1997) assume that the expected rate of return of the portfolio is zero. The main reason is the relative high value of the portfolio volatility. The difficulty of obtaining a good quality estimate for the rate of return of the portfolio is another argument.

VaR ESTIMATES FOR PORTFOLIO WITH LONG (OR SHORT) POSITIONS ONLY

For a given (1-α)% confidence level (e.g., 95%), assuming a normal probability distribution function for the rates of return, the VaR of a given portfolio is computed as (Longerstaey and Spencer 1996; page 8):

$$\begin{aligned} VaR &= Portfolio \cdot [1 - \exp(\mu \cdot t - z \cdot \sigma \cdot \sqrt{t})] \cong \\ &\cong Portfolio \cdot (-\mu \cdot t + z \cdot \sigma \cdot \sqrt{t}) \\ &\cong Portfolio \cdot z \cdot \sigma \cdot \sqrt{t} \end{aligned} \qquad (10)$$

where:

Portfolio	=	portfolio value
μ	=	portfolio growth rate
t	=	time horizon
σ	=	portfolio volatility
z	=	the (1-α) quantile of the standard normal distribution N(0,1)

It should be noted that using the **z** value is correct only when the portfolio volatility is known exactly. If an estimate of the portfolio volatility is used instead of the true value of the portfolio volatility, than a correct estimate of VaR requires the use of the Student **t** value. The Student **t** value is larger than the corresponding **z** value. The VaR based on the Student **t** value is larger than the VaR based on the corresponding **z** value. This fact accounts for the uncertainty generated as a result of using an estimate of the portfolio volatility instead of the true value of the portfolio volatility. However, when the portfolio volatility is estimated using a large number of data (e.g., greater than 30), the Student **t** value is practically equal to the **z** value.

Based on the volatilities of the individual assets ($\boldsymbol{\sigma_i}$), their weights in the portfolio value ($\mathbf{w_i}$), and their correlation coefficients ($\boldsymbol{\rho_{ij}}$), the portfolio volatility is computed as (Jorion 1997; page 150):

$$\sigma = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{ij} \cdot w_i \cdot w_j \cdot \sigma_i \cdot \sigma_j} \qquad (11)$$

It should be noted that the weights $\mathbf{w_i}$ cannot be computed for a portfolio with both long and short positions having a total net value of zero. Moreover, a portfolio with both long and short positions having a total value close to zero may have a relatively high volatility (McCarthy 1999).

VaR ESTIMATES FOR PORTFOLIO WITH BOTH LONG AND SHORT POSITIONS

Based on the RiskMetrics approximations for continuously compounded returns and VaR, it results that:

$$\begin{aligned} VaR = & -\sum_{i=1}^{N} Asset_i \cdot \mu_i \cdot t + \\ & + z \cdot \sqrt{t} \cdot \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{ij} \cdot Asset_i \cdot Asset_j \cdot \sigma_i \cdot \sigma_j} \\ & \cong z \cdot \sqrt{t} \cdot \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{ij} \cdot Asset_i \cdot Asset_j \cdot \sigma_i \cdot \sigma_j} \end{aligned} \qquad (12)$$

where:

$\mathbf{Asset_i}$	=	portfolio value invested in asset **i**
$\boldsymbol{\mu_i}$	=	the growth rate for asset **i.**

A positive value for $\mathbf{Asset_i}$ denotes a long position, while a negative value denotes a short position.

This approximation (equation 12) is similar to equation (1.39) of Deutsch (2003). It should be noted that equation (1.39) of Deutsch (2003) is for portfolios having either long or short positions. If **t** = 1 equation (12) becomes identical with equation (10) of Chapados and Bengio (2000). TotalSum (2003) provides a numerical example for a portfolio having both long and short positions using equation (12) as a VaR approximation with $\boldsymbol{\mu_i} = 0$.

The corresponding "un-diversified" VaR is:

$$VaR_{undiversified} = -\sum_{i=1}^{N} Asset_i \cdot \mu_i \cdot t + + z \cdot \sqrt{t} \cdot \sum_{i=1}^{N} \cdot |Asset_i| \cdot \sigma_i \qquad \cong$$

$$\cong z \cdot \sqrt{t} \cdot \sum_{i=1}^{N} \cdot |Asset_i| \cdot \sigma_i \qquad (13)$$

The VaR approximation (equation 12) handles in a smooth manner the case of portfolios having an aggregate value close to zero.

As long as the direct effect of the rates of return is neglected for VaR estimates (i.e., $\boldsymbol{\mu_i} = 0$), the long and short holders of a given portfolio have the same risk. If the total amount (i.e., long and short positions) invested in each asset (or assets having exactly the same volatility and correlation coefficient +1.0) is zero, then the VaR is zero. Similarly, if all positions in each asset are matched by equal positions in another asset that has exactly the same volatility and the correlation coefficient of these two assets is exactly –1.0, then the VaR is zero. When the hedging is done using assets that (1) are not perfectly (positively or negatively) correlated and (2) have different volatilities, then the VaR is greater than zero, even when the total portfolio value is zero.

If the growth rates are taken into account, this VaR approximation accounts for an increased risk due to (1) long positions with negative growth rates or (2) short positions with positive growth rates. Similarly, it accounts for a reduced risk due to (1) long positions with positive growth rates or (2) short positions with negative growth rates.

MONTE CARLO VALUE AT RISK ESTIMATES FOR PORTFOLIO WITH BOTH LONG AND SHORT POSITIONS

All the long positions can be lumped together into an "index." Based on historical data for the individual stocks, the history of this "index" can be established. Similarly, all short positions can be lumped together into another "index" and its history can be established from the historical data available for the individual stocks. For each one of these "indexes" we can compute its growth rate and volatility. Additionally, we can compute the correlation coefficient between the rates of return of the two "indexes." The Monte Carlo simulation is required only for two positions, i.e. the two "indexes." The actual return rates for these "indexes" are generated based on correlated random numbers. To be compatible with the VaR estimates provided by equations (10) and (12), the Monte Carlo simulation must assume the normality of the return rates for these "indexes." (Using a historical approach for Monte Carlo simulations may be required when we are faced with significant departures from normality. However, this case is outside the scope of our analysis.)

When the portfolio contains only long (or short) positions, the Monte Carlo VaR estimate is close to the VaR estimate provided by equation (10) (without applying the approximation for the exponential function provided by equation (9)). When the portfolio contains both long and short

positions resulting in an overall portfolio value close to zero, the Monte Carlo VaR estimate is close to the VaR estimate provided by equation (12). Therefore, for portfolios containing both long and short positions, in order to check the numerical effects of different approximations, it is recommended to use both the Monte Carlo method and equation (12). For portfolios containing only long (or short) positions, the VaR estimate provided by equation (10) is preferable to the estimates provided by equation (12) or Monte Carlo simulation.

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