SBP VALUATION
TSR REINVESTMENT OF DIVIDEND DISTRIBUTIONS

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ABSTRACT

We consider a portfolio containing shares (of a given company) that is reinvesting immediately (in the stock of the same company) all dividend distributions. As a result of the immediate reinvestment of the dividend distributions, the growth rate for the value of the portfolio is the stock growth rate when there are no dividend distributions. This result may be useful for the valuation of TSR contracts.

INTRODUCTION

Suppose that S, the stock price of company A, grows at a constant growth rate, μ. Suppose that company A does not distribute any dividends. Under deterministic conditions (i.e., zero stock volatility) the stock dynamics can be described by the following finite differences (Jorion 2001) and ordinary differential equations (Hull 1993):

$$S_{t+\Delta t} = S_t + \mu \cdot S_t \cdot \Delta t \quad ; \quad S(t_0) = S_0 ; \quad \Delta t \to 0$$

$$\frac{dS}{dt} = \mu \cdot S \quad ; \quad S(t_0) = S_0$$

Under stochastic conditions (i.e., $\mu = \mu_{avg} + \mu'$ where μ' is a non-correlated normally distributed perturbation with zero average) the stock dynamics can be described by the following stochastic differential equation (Hull 1993):

$$dS = \mu_{avg} \cdot S \cdot dt + \sigma \cdot S \cdot dW \quad ; \quad S(t_0) = S_0$$

MODELING OF DIVIDEND DISTRIBUTIONS

Suppose that company A does distribute dividends with a constant continuous time dividend yield y. Let us consider a portfolio containing N shares (of company A) that is investing immediately all dividend distributions in the stock of company A. With these assumptions the portfolio value, P, is described by the following equations:

- ***Finite Differences Equations:***

$$S_{t+\Delta t} = S_t + (\mu - y) \cdot S_t \cdot \Delta t \quad ; \quad S(t_0) = S_0 \, ; \quad \Delta t \to 0$$
$$N_{t+\Delta t} = N_t + y \cdot N_t \cdot \Delta t \quad ; \quad N(t_0) = N_0$$
$$P_{t+\Delta t} = S_{t+\Delta t} \cdot N_{t+\Delta t}$$

- ***Ordinary Differential Equations:***

$$\frac{dS}{dt} = (\mu - y) \cdot S \quad ; \quad S(t_0) = S_0$$
$$\frac{dN}{dt} = y \cdot N \quad ; \quad N(t_0) = N_0$$
$$P = S \cdot N$$

- ***Stochastic Differential Equations:***

$$dS = (\mu_{avg} - y) \cdot S \cdot dt + \sigma \cdot S \cdot dW \quad ; \quad S(t_0) = S_0$$
$$dN = y \cdot N \cdot dt \quad ; \quad N(t_0) = N_0$$
$$P = S \cdot N$$

THE EFFECT OF DIVIDEND DISTRIBUTIONS

Finite Differences Equations:
The solution of the Finite Differences Equations is:

$$S_{i \cdot \Delta t} = [1 + (\mu - y) \cdot \Delta t]^i \cdot S_0$$
$$N_{i \cdot \Delta t} = (1 + y \cdot \Delta t)^i \cdot N_0$$
$$P_{i \cdot \Delta t} = [1 + (\mu - y) \cdot \Delta t]^i \cdot (1 + y \cdot \Delta t)^i \cdot N_0 \cdot S_0$$

It should be noted that for:

$$i \cdot \Delta t = t \quad ; \quad \Delta t \to 0$$

We obtain:

$$[1 + (\mu - y) \cdot \Delta t]^i \cdot (1 + y \cdot \Delta t)^i \to e^{(\mu - y) \cdot t} \cdot e^{y \cdot t}$$
$$P \to e^{\mu \cdot t} \cdot N_0 \cdot S_0$$

Therefore, for small time intervals, the growth rate for the value of the portfolio is the stock growth rate when there are no dividend distributions.

Ordinary Differential Equations:
The solution of the ordinary differential equations is:

$$S = S_0 \cdot e^{(\mu - y) \cdot t}$$
$$N = N_0 \cdot e^{y \cdot t}$$
$$P = S_0 \cdot N_0 \cdot e^{\mu \cdot t}$$

Therefore, under deterministic conditions, the growth rate for the value of the portfolio is the stock growth rate when there are no dividend distributions.

Stochastic Differential Equations:
Under stochastic conditions we investigate the dynamics of the value of the portfolio:

$$P = S \cdot N = S \cdot N_0 \cdot e^{y \cdot t}$$

Applying the Ito's lemma (1944; 1946; 1951) we obtain the following:

$$\frac{\partial P}{\partial S} = N_0 \cdot e^{y \cdot t}; \quad \frac{\partial^2 P}{\partial S^2} = 0; \quad \frac{\partial P}{\partial t} = y \cdot S \cdot N_0 \cdot e^{y \cdot t};$$
$$dP = [(\mu_{avg} - y) \cdot S \cdot N_0 \cdot e^{y \cdot t} + y \cdot S \cdot N_0 \cdot e^{y \cdot t}] \cdot dt +$$
$$+ \sigma \cdot S \cdot N_0 \cdot e^{y \cdot t} \cdot dW;$$

After simplifications we obtain:

$$dP = \mu_{avg} \cdot P \cdot dt + \sigma \cdot P \cdot dW; \quad P(t_0) = P_0$$

Therefore, under stochastic conditions, the growth rate for the value of the portfolio is the stock growth rate when there are no dividend distributions. Moreover, the volatility of the value of the portfolio is the volatility of the stock of the company.

CONCLUSIONS

As a result of the immediate reinvestment of the dividend distributions, the growth rate for the value of the portfolio is the stock growth rate when there are no dividend distributions.

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