REAL OPTIONS – CAPITAL BUDGETING

Sorin R. Straja, Ph.D., FRM
Montgomery Investment Technology, Inc.
200 Federal Street
Camden, NJ 08103
Phone: (610) 688-8111
sorin.straja@fintools.com
www.fintools.com

Most investment decisions present the following three characteristics:
- *Irreversibility*: The investment is partially or completely sunk, i.e. it cannot be recovered.
- *Uncertainty*: The best that can be done is to assess the probabilities of the alternative outcomes that can imply larger or smaller profits.
- *Timing*: The decision can be postponed in order to get more information (but never complete information) about future alternatives.

The classical theory has not addressed the qualitative and quantitative aspects resulting from the interaction of the above mentioned characteristics. The Marshallian theory computes the ratio between the present value of the profits (**V**) and the present value of the investments (**I**). Whenever this ratio exceeds the unit (i.e. whenever the net present value of the project (**V - I**) is greater than zero), it means the investment should be done. In reality, a firm having to decide about a given investment has an "option" analogous to a call option: it has the right to invest, but it has no obligation to invest. When the firm decides to invest, it makes an irreversible investment expenditure, therefore it exercises, or "kills", its option to invest. It gives up the possibility of waiting for additional information that might affect the desirability or timing of the expenditure, and it cannot disinvest should market conditions change adversely. This lost option value is a cost that should be included as part of the total cost of investment. Therefore the Marshallian rule should be modified, in the sense that the critical ratio to be exceeded should not be equal to the unit, but greater than the unit.

Usually, there is a high degree of uncertainty concerning the present value of the profits, **V**, while the present value of the investments, **I**, is known with an acceptable degree of accuracy. Therefore, most models investigate the dynamics of **V** using a stochastic Brownian process. The parameters of the stochastic process are $\boldsymbol{\alpha}$ (the drift parameter), $\boldsymbol{\sigma}$ (the variance parameter). Using $\boldsymbol{\rho}$ (the discount rate), the main output of the model is the critical ratio: the higher the uncertainty, the higher the critical ratio. Details of the model are presented in the Appendix.

As a first approximation, $\boldsymbol{\sigma}$ can be taken as being equal to the volatility of the market where the investment is supposed to take place. As an example, the volatility of the NYSE can be used as a first guess, or better the volatility of that particular industry where the investment should be done. The drift parameter is a direct reflection of the timing effect: there are cases when the later the investment is done, the lower the profits, or vice-versa. A simple way to estimate both $\boldsymbol{\alpha}$ and $\boldsymbol{\sigma}$ is to use the present values (**V**) computed for different investment moments, each time taking advantage of the additional information that becomes available.

Capital Budgeting Examples Workbook of FinTools.com

Example 1: Company XYZ has to decide about a new project. The investment cost is $80 M. The expected profits, by the end of each year of the five-year life of the project, are $39 M, $30 M, $21 M, $37 M and $46 M, respectively. The discount rate is 10%, the finance rate is 10%, and the reinvestment rate is 14%. All rates are annualized rates.

As an additional effort, in order to take into account the uncertainties, the company has to estimate the two parameters of the stochastic model: $\boldsymbol{\alpha}$ (the drift parameter), and $\boldsymbol{\sigma}$ (the variance parameter). As a first approximation, it is decided to discard the effect of the drift parameter ($\boldsymbol{\alpha} = 0$), and to approximate the variance parameter with the typical NYSE value ($\boldsymbol{\sigma} = 20\%$).

The inputs and outputs of the software are listed in Table 1. We mention that the investment is a negative number, because it represents an amount that is always spent. The output section lists the present values of the profits ("Other cash flows") and investments (always a negative number). The net present value of the project is the algebraic sum of the net present values of the profits and investments. The internal rate of return and the modified internal rate of return are computed according to the usual standards. Based upon the internal rate of return (≈32%) which exceeds the discount rate of 10%, we decide that the XYZ company should invest. Using the Marshallian theory, because the net present value of the project is positive (≈$50 M), we reach the same conclusion.

The above conclusions disregarded the uncertainties associated with the estimated profits ("Other cash flows"). In order to include the uncertainties in the decision process, the model computes the value of the option to delay the investment (≈$46 M) and the critical profitability index (1.57).

Using an extensive approach, at moment zero, before any decision is taken, the company has ≈$126 M representing the sum of intended investment and of the option to delay investment ($80 M + $46 M). This amount represents the critical present value of the profits ("Other cash flows") in the sense that no investment should be done as long as the present value of the profits ("Other cash flows") is less than the critical present value of the profits ("Other cash flows"). In this particular case the present value of the profits ("Other cash flows") is ≈$130 M, it is larger than the critical present value of the profits ("Other cash flows") of ≈$126 M, and therefore the XYZ company should invest.

Similarly, using an intensive approach, the profitability index is the ratio between the present value of the profits ("Other cash flows") and investments. In order to make an investment, the profitability index should be larger than the critical profitability index computed by the model. In this particular case the profitability index is 1.62, it is larger than the critical profitability index of 1.57, and therefore the XYZ Company should invest.

We point out that the extensive and intensive approaches are equivalent, and therefore they always provide the same final conclusion.

Table 1.

FinTools® OPTIONS XL Capital Budgeting

Input:

			Cash Flow Range		
				Cash Flows	
Function (below)		Time (yr)		Investment	Operating
Drift (Alpha)	0.00%	0		-80.00	
Sigma (Volatility)	20.00%	1			39.00
Type of Discount Rate (0=ann, 1=cont)	0	2			30.00
Discount Rate	10.00%	3			21.00
Cash Flow Range (right)		4			37.00
Finance Rate	10.00%	5			46.00
Reinvestment Rate	14.00%				

Output:

Function: CapitalProject()

Classical Model	Func	
Present Value of "Operating Flows"	1	129.86
Present Value of "Investment Flows"	2	-80.00
Net Present Value of the Project	3	49.86
Internal Rate of Return	4	0.32
Modified Internal Rate of Return	5	0.23
Stochastic Model - Extensive Approach		
Critical Value of the Option to Delay Investment	6	45.99
Critical Present Value of "Operating Flows"	7	125.99
Stochastic Model - Intensive Approach		
Profitability Index	8	1.62
Critical Profitability Index	9	1.57

Function: CapitalDecision()

Internal Rate of Return	1	Invest
Modified Internal Rate of Return	2	Invest
Marshallian Theory	3	Invest
Dixit & Pindyck	4	Invest

Example 2: Company XYZ has to decide about a new project. The investment cost is \$15 M. The expected profits, by the end of each year of the five-year life of the project, are \$5.00 M, \$5.25 M, \$5.51 M, \$5.79 M and \$6.08 M, respectively. The discount rate is 10%, the finance rate is 10%, and the reinvestment rate is 14%. All rates are continuous rates.

As an additional effort, in order to take into account the uncertainties, the company has to estimate the two parameters of the stochastic model: $\boldsymbol{\alpha}$ (the drift parameter), and $\boldsymbol{\sigma}$ (the variance parameter). As a first approximation, it is decided to discard the effect of the drift parameter ($\boldsymbol{\alpha}$ = 0), and to approximate the variance parameter with the typical volatility value of the industry ($\boldsymbol{\sigma}$ = 30%).

The inputs and outputs of the software are listed in Table 2. We mention that the investment is a negative number, because it represents an amount that is always spent. The output section lists the present values of the profits ("Other cash flows") and investments (always a negative number). The net present value of the project is the algebraic sum of the net present values of the profits and investments. The internal rate of return and the modified internal rate of return are computed according to the usual standards. Based upon the internal rate of return (≈21%) which exceeds the discount rate of 10%, we decide that the XYZ Company should invest. Using the Marshallian theory, because the net present value of the project is positive (≈\$5.5 M), we reach the same conclusion.

The above conclusions disregarded the uncertainties associated with the estimated profits ("Other cash flows"). In order to include the uncertainties in the decision process, the model computes the value of the option to delay the investment (≈\$14 M) and the critical profitability index (1.93).

Using an extensive approach, at moment zero, before any decision is taken, the company has ≈\$29 M representing the sum of intended investment and of the option to delay investment (\$15 M + \$14 M). This amount represents the critical present value of the profits ("Other cash flows") in the sense that no investment should be done as long as the present value of the profits ("Other cash flows") is less than the critical present value of the profits ("Other cash flows"). In this particular case the present value of the profits ("Other cash flows") is ≈\$20 M, it is smaller than the critical present value of the profits ("Other cash flows") of ≈\$29 M, and therefore the XYZ company should not invest.

Similarly, using an intensive approach, the profitability index is the ratio between the present value of the profits ("Other cash flows") and investments. In order to make an investment, the profitability index should be larger than the critical profitability index computed by the model. In this particular case the profitability index is 1.36, it is smaller than the critical profitability index of 1.93, and therefore the XYZ company should not invest.

We point out that the extensive and intensive approaches are equivalent, and therefore they always provide the same final conclusion.

Table 2.

FinTools® OPTIONS XL Capital Budgeting

Input:

		Time (yr)	Investment	Operating
Function (below)				
Drift (Alpha)	0.00%	0	-15.00	
Sigma (Volatility)	30.00%	1		5.00
Type of Discount Rate (0=ann, 1=cont)	1	2		5.25
Discount Rate	10.00%	3		5.51
Cash Flow Range (right)		4		5.79
Finance Rate	10.00%	5		6.08
Reinvestment Rate	14.00%			

(Cash Flow Range — Cash Flows: Investment, Operating)

Output:

Function: CapitalProject()

Classical Model	Func	
Present Value of "Operating Flows"	1	20.47
Present Value of "Investment Flows"	2	-15.00
Net Present Value of the Project	3	5.47
Internal Rate of Return	4	0.21
Modified Internal Rate of Return	5	0.18
Stochastic Model - Extensive Approach		
Critical Value of the Option to Delay Investment	6	13.99
Critical Present Value of "Operating Flows"	7	28.99
Stochastic Model - Intensive Approach		
Profitability Index	8	1.36
Critical Profitability Index	9	1.93

Function: CapitalDecision()

Internal Rate of Return	1	Invest
Modified Internal Rate of Return	2	Invest
Marshallian Theory	3	Invest
Dixit & Pindyck	4	Do not invest

Example 3: Company XYZ has to decide about a new project. The investment cost is \$15 M. The expected profits, by the end of each year of the five-year life of the project, are \$5.00 M, \$5.25 M, \$5.51 M, \$5.79 M and \$6.08 M, respectively. The discount rate is 10%, the finance rate is 10%, and the reinvestment rate is 14%. All rates are continuous rates.

As an additional effort, in order to take into account the uncertainties, the company has to estimate the two parameters of the stochastic model: **α** (the drift parameter), and **σ** (the variance parameter).

The company decides to ask seven different consultants to estimate the present value of the profits. Each consultant receives another date as the moment to start the potential investment as follows: 1/2/87, 1/3/88, 1/4/89, 1/5/89, 1/6/89, 1/7/90, 1/8/90. The present values of the profits, reported by the seven consultants are \$20.00 M, \$20.13 M, \$19.88 M, \$20.00 M, \$20.50 M, \$20.25 M, and \$20.88 M, respectively.

Using these data as input, the software estimates **α** (the drift parameter) and **σ** (the variance parameter). The results are listed in Table 3a. The 95% confidence level for **α** (the drift parameter) is from -43% to +57%, its maximum likelihood estimate being 7%. Therefore the effect of the drift parameter can be disregarded choosing **α** = 0 . The variance parameter is 34%. These values are now used in order to make a decision about the investment.

Table 3a.

FinTools® OPTIONS XL Capital Budgeting: Parameters Identification

Input:				
Function (below)				
Confidence Level		95%		
OperatingFlowRange (right)		Investment	PV of	
Investment Date		Date	Operating Flows	
PV of Operating Flows		1/2/1987	20.00	
		1/3/1988	20.13	
		1/4/1989	19.88	
		1/5/1989	20.00	
		1/6/1989	20.50	
		1/7/1990	20.25	
		1/8/1990	20.88	
Output:	Func	**Per year**		
Drift	1	7.16%		
Drift Lower Limit	2	-42.96%	Based on the specified Confidence Level	
Drift Upper Limit	3	57.28%		
Sigma	4	33.88%		
Standard Error of Sigma	5	9.78%	24.10%	43.66%

The inputs and outputs of the software are listed in Table 3b. We mention that the investment is a negative number, because it represents an amount that is always spent. The output section lists the present values of the profits ("Other cash flows") and investments (always a negative number). The net present value of the project is the algebraic sum of the net present values of the profits and investments. The internal rate of return and the modified internal rate of return are computed according to the usual standards. Based upon the internal rate of return (≈21%) which exceeds the discount rate of 10%, we decide that the XYZ company should invest. Using the Marshallian theory, because the net present value of the project is positive (≈$5 M), we reach the same conclusion.

Table 3b.

FinTools® OPTIONS XL Capital Budgeting

Input:

Function (below)	
Drift (Alpha)	0.00%
Sigma (Volatility)	34.00%
Type of Discount Rate (0=ann, 1=cont)	1
Discount Rate	10.00%
Cash Flow Range (right)	
Finance Rate	10.00%
Reinvestment Rate	14.00%

Cash Flow Range

	Cash Flows	
Time (yr)	Investment	Operating
0	-15.00	
1		5.00
2		5.25
3		5.51
4		5.79
5		6.08

Output:

Function: CapitalProject()

Classical Model	Func	
Present Value of "Operating Flows"	1	20.47
Present Value of "Investment Flows"	2	-15.00
Net Present Value of the Project	3	5.47
Internal Rate of Return	4	0.21
Modified Internal Rate of Return	5	0.18
Stochastic Model - Extensive Approach		
Critical Value of the Option to Delay Investment	6	16.54
Critical Present Value of "Operating Flows"	7	31.54
Stochastic Model - Intensive Approach		
Profitability Index	8	1.36
Critical Profitability Index	9	2.10

Function: CapitalDecision()

Internal Rate of Return	1	Invest
Modified Internal Rate of Return	2	Invest
Marshallian Theory	3	Invest
Dixit & Pindyck	4	Do not invest

The above conclusions disregarded the uncertainties associated with the estimated profits ("Other cash flows"). In order to include the uncertainties in the decision process, the model computes the value of the option to delay the investment (≈$16 M) and the critical profitability index (2.10).

Using an extensive approach, at moment zero, before any decision is taken, the company has ≈$31 M representing the sum of intended investment and of the option to delay investment ($15 M + $16 M). This amount represents the critical present value of the profits ("Other cash flows") in the sense that no investment should be done as long as the present value of the profits ("Other cash flows") is less than the critical present value of the profits ("Other cash flows"). In this particular case the present value of the profits ("Other cash flows") is ≈$20 M, it is smaller than the critical present value of the profits ("Other cash flows") of ≈$31 M, and therefore the XYZ company should not invest.

Similarly, using an intensive approach, the profitability index is the ratio between the present value of the profits ("Other cash flows") and investments. In order to make an investment, the profitability index should be larger than the critical profitability index computed by the model. In this particular case the profitability index is 1.36, it is smaller than the critical profitability index of 2.10, and therefore the XYZ Company should not invest.

We point out that the extensive and intensive approaches are equivalent, and therefore they always provide the same final conclusion.

APPENDIX

Stochastic model.

$$dV = \alpha V \, dt + \sigma V \, dz$$

$$F = \text{Max} \; E\left(\left[V(t) - I \right] \cdot e^{-\rho t}, 0 \right)$$

The stochastic model takes into account the fluctuations of V in time; dz is the Wiener process with zero expectation and variance equal to dt; the process has a drift and a variance parameter. The second equation defines the value of the option to delay investment (or the investment opportunity) and requires its maximization. The best moment to invest is T, and the corresponding value of V = V(T) is denoted by V^*. Investment should be done when $V > V^*$.

Dynamic programming

The Bellman's equation becomes:

$$\frac{1}{2}\sigma^2 V^2 F'' + \alpha V F' - \rho F = 0 \quad for \quad V < V^*$$

$$F = V - I \quad for \quad V > V^*$$

The following equations set the boundary conditions for F; the last two conditions implement the matching and smooth pasting requirements:

$$F(0) = 0$$

$$F(V^*) = V^* - I$$

$$\frac{dF}{dV}\Big|_{V=V^*} = 1$$

Deterministic model

The deterministic model is a particular case of the stochastic one. It is obtained from the stochastic model for $\sigma = 0$.

Marshallian model

The neoclassical Marshallian model is a particular case of the deterministic model. It is obtained from the deterministic one setting $\alpha = 0$.

Notations

E	=	Expected value
V	=	Present value of the profits ($)
I	=	Sunk cost (investment) of the project ($)
F	=	Value of the investment opportunity (option to delay investment) ($)
t	=	current time (years)
T	=	time to invest (years)
α	=	drift parameter (1/year)
ρ	=	discount rate (1/year)
σ	=	variance parameter ($1/\sqrt{\text{year}}$)