EMPLOYEE STOCK PURCHASE PLAN

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The **ESPP** contract is a special type of European call option. The strike is the minimum between $\mathbf{S_0 * (1 - d)}$ and $\mathbf{S * (1 - d)}$, where:

$\mathbf{S_0}$ is the underlying today,
S is the underlying on the exercise date, and
d is a discount rate (between 0% and 100%)

Therefore, the payoff of this contract (on the expiration date) is:

Payoff **ESPP** = $\mathbf{S * d}$	if	$\mathbf{S \le S_0}$
Payoff **ESPP** = $\mathbf{S - S_0 * (1 - d)}$	if	$\mathbf{S > S_0}$

Additionally, let us consider:

1. Contract **A** which is a European call option with the strike price equal to $\mathbf{S * (1 - d)}$ for any value of the underlying; and
2. Contract **B** which is a European call option with the strike equal to $\mathbf{S_0}$.

Therefore,

Payoff **A** = $\mathbf{S * d}$	if	$\mathbf{S \le S_0}$
Payoff **A** = $\mathbf{S * d}$	if	$\mathbf{S > S_0}$
Payoff **B** = **0**	if	$\mathbf{S \le S_0}$
Payoff **B** = $\mathbf{S - S_0}$	if	$\mathbf{S > S_0}$

Let us consider a portfolio **C** containing one contract **A** and $\mathbf{(1 - d)}$ contracts **B**. The payoff of portfolio **C** is:

Payoff **C** = $\mathbf{S * d}$	if	$\mathbf{S \le S_0}$
Payoff **C** = $\mathbf{S * d + (1 - d) * (S - S_0)}$	if	$\mathbf{S > S_0}$

But,

$$\mathbf{S * d + (1 - d) * (S - S_0) = S * d + S - S_0 - d * S + d * S_0 = S - S_0 * (1 - d)}$$

Therefore, the payoff of portfolio **C** is identical with the payoff of the **ESPP** contract, and the value of the **ESPP** contract is equal to the value of one contract **A** and **(1 – d)** contracts **B**. It should be pointed out the value of contract **B** is provided by the usual Black-Scholes formula, while for contract **A** we have:

$$\text{Value } \mathbf{A} = \mathbf{S_0 * \exp(- y * T) * d}$$

where:

y is the yield rate, and
T is the time until expiration.

It can be easily seen that a portfolio with **exp(- y * T) * d** shares of the underlying today will have **d** shares of underlying on the expiration date, and therefore its value is always equal to the payoff of contract **A**. Formally, it can be demonstrated through direct verification that the valuation provided for contract **A** satisfies the Black-Scholes equation with the condition:

$$\text{Option value} = \mathbf{S * d} \qquad \text{on the expiration date.}$$