ESO VALUATION – INGERSOLL MODEL

Sorin R. Straja, Ph.D., FRM
Montgomery Investment Technology, Inc.
200 Federal Street
Camden, NJ 08103
Phone: (610) 688-8111
sorin.straja@fintools.com
www.fintools.com

INTRODUCTION

The employee stock options are significantly different with respect to the usual traded options. FASB (2004) acknowledges that the main difference is the asymmetric relationship between the writer of the option (i.e., the company) and the holder of the option (i.e., the manager):

> "A26. The fair value of a traded (or transferable) share option is based on its contractual term because rarely is it economically advantageous to exercise, rather than sell, a transferable share option before the end of its contractual term. Employee share options generally differ from transferable share options in that employees cannot sell (or hedge) their share options—they can only exercise them; because of this, employees generally exercise their options before the end of the options' contractual term. Thus, the inability to sell or hedge an employee share option effectively reduces the option's value because exercise prior to the option's expiration terminates its remaining life and thus its remaining time value."

The standard finance theory strongly recommends diversification of holdings to minimize the risk. However, the employees, managers, and executives cannot diversify their portfolios that usually have large holdings of their firms stock. There are many reasons for such undiversified holdings. The stock in question may be in a pension or profit sharing plan over which the employee has no control, or it may be phantom or restricted stock or incentive options that cannot be sold. There might be a legal restriction on selling the firms stock. Some executives' contracts require large holdings of the company's stock. Less explicitly, the restriction may be due to a large capital gain that the manager is unwilling to realize, or the manager may simply feel "morally" constrained not to sell his company's stock. Due to the above reasons, hedging of these positions is not possible, and therefore the portfolios are not adequately diversified. Because managers hold undiversified portfolios, their stock ownership and equivalent items such as incentive options have a subjective value to them that is less than their market value.

The Ingersoll (2002) model is simply the Black-Scholes model with modified parameters. This model (Ingersoll 2002) provides three estimates regarding the employee stock option:

1. The **Subjective Value** of the contract. The Subjective Value is the value attached by the constrained manager.

2. The **Objective Value** of the contract. The Objective Value corresponds to the behavior of the employee assessed from the market perspective.

3. The **Market Value** of the contract. The Market Value is the value the option would have if held by an unconstrained agent.

Ingersoll (2002) uses a power utility function approach to evaluate the Subjective Value and the employee behavior. Because of the restrictions imposed on the manager, the Subjective Value is less than the Market Value, and the exercise behavior of the manager appears to be sub-optimal from the market perspective.

The Ingersoll (2002) model requires three additional parameters: the standard deviation of the residuals (from the Capital Asset Pricing Model (CAPM)); the fraction of the portfolio wealth (required in that company stock); and the Coefficient of Relative Risk Aversion (CRRA).

The default value for CRRA is 2. This value is used in a variety of other studies in the economics literature (Feldstein and Ranguelova 2001; Milevsky and Panyagometh 2001; Browne et al. 2003) that "peg" the Coefficient of Relative Risk Aversion at approximately 2.

SYNTAX
PR_OptionsIngersoll_EB(Model, TypeOpt, Func, Underlying, Exercise, ValueDate, VestDate, ExpirationDate, Volatility, InterestRate, YieldRate, LatticeType, StdevResidual, WealthCompany, RiskAversion, Exit_PreVest, Exit_PostVest)

The following arguments are required by the model:

Model is the model setting:
This specifies the number of iterations to be performed. The iterations must be ≥ 2 and ≤ 2500.

TypeOpt is the option type:
1 = "C" = call

Func is the desired output. Enter '201'. This gives an array of theoretical and fugit values.

Underlying is the price of the underlying asset. The price must be > 0.

Exercise is the price at which the asset can be purchased. Exercise is also referred to as the strike price. The exercise price must be > 0.

Value Date is the date the valuation is done.

Vesting Date is the date when the stock option vests.

Expiration Date is the date when the stock option expires.

Volatility is the annualized volatility of the underlying asset price. Volatility is defined as the standard deviation of the relative price changes. Volatility must be > 0.

Interest Rate is the prevailing risk-free interest rate expressed as a percentage. The interest rate must be ≥ 0.

Yield Rate is the yield expressed as a percentage (dividends or interest yield) from the underlying asset. The yield rate must be ≥ 0.

Lattice Type is the type of lattice intended to be used.
Use the following inputs:

0	Black-Scholes
1	Ingersoll European
2	Binomial
2.1	Binomial / Ingersoll European / Richardson
2.2	Binomial / Ingersoll European
2.5	Binomial / Average (1/2 Step)
3	Trinomial

Standard Deviation of the Residuals is the residual volatility of the market. It is calculated from Capital Asset Pricing Model (CAPM). The standard deviation of the residuals must be > 0.

Wealth in Company Stock is the fraction of the total wealth invested in the underlying company. The fraction of total wealth invested in the underlying company must be ≥ 0% and ≤ 100%.

Coefficient of Relative Risk Aversion is generally equal to 2.

Exit Rate Pre-Vesting is the turnover or forfeiture rate prior to vesting.

Exit Rate Post-Vesting is the turnover or forfeiture rate post vesting. A payoff is assumed for an in-the-money option.

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