Effect of Time to Exercise Distribution on the Fair Value of Employee Stock Options

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Introduction

In an earlier White Paper[1] on the valuation of Employee Stock Options (ESOs) we demonstrated that exercise behavior can have a significant effect on the calculated fair value. Assumptions concerning exercise behavior are divided into two general categories. The first involves an explicit assumption regarding the elapsed time between vesting and exercise. The SAB 107 Simplified Method[2] is an excellent example. All outstanding vested options are assumed to exercise at a time which is midway between the vesting date and the expiration date of the options. One may assume that the options are exercised upon immediately upon vesting, or at some other time. Such assumptions are very convenient because the well-known, closed-form Black-Scholes-Merton method may be used to calculate the fair value of the options. Since the BSM method is valid only for European-style options, a fixed time of exercise allows the valuator to employ the method.

The other category assumes that the holder will exercise the options if and when the value of the underlying shares exceeds a predetermined threshold. The ratio of the threshold value to the share value on the award date is called a *Suboptimal Exercise Factor*. Typically, some form of lattice-based computational method is used, since the option must be modeled as American-style.

ASC 718[3] offers guidelines for the use of such assumptions. Depending upon the particular assumption chosen, the resulting fair value can vary over a wide range. In this article, we examine the first category of ESO expiration assumptions and comment upon how a particular assumption may be developed to either represent an issuer's past experience with similar awards or to produce a fair value which satisfies a particular objective or objectives of the Finance Committee or Board of Directors.

The Basic Model

To illustrate our basic model, we shall make use of the test case presented in FAS 123R Illustration 4.[4]

The assumptions and information about the options are as follows:

Stock Price @ Grant Date	**$30.00**	**Risk-free Rate over CT**	**1.5 to 4.3%**
Exercise Price	**$30.00**	**Expected Volatility over CT**	**40 to 60%**
Contractual Term	**10.0 years**	**Expected Dividend Yield**	**1.0%**
Vesting Term	**3.0 years**	**Suboptimal Exercise Factor**	**2.0**

Using a lattice-based computational method with simplifying assumptions of constant volatility (50%) and constant risk-free rate (2.9%), the fair value can be calculated as $14.69 per unit. This result corresponds to a European-style option with identical conditions, save for a contractual term of 7.0 years. Using this expected term, the fair value of $14.69 is calculated using the Black-Scholes-Merton method.

We now consider the effect of expected term upon the fair value of the option as envisioned in Illustration 4. Exhibit 1, ***Base Case,*** presents the fair value of the call option in Illustration 4 over expected terms from 3.00 to 10.00 years at various volatilities from 20% to 80%. The Black-Scholes-Merton method was used to calculate the fair values. Note that fair value is a monotonically increasing function of both term and volatility. For a volatility of 50%, the fair value of the option varies from

$10.31 at three years (exercise immediately upon vesting) to $16.58 if the option is exercised on the date of expiration. To value an option of this kind, the expected term must be specified as an input variable.

In a real-world situation, recipients of employee stock options will exercise them at different times for a variety of different reasons. Some recipients will forfeit their options due to a change of employment. Still others may simply neglect to exercise options and let them expire. The assumption of an expected term or a suboptimal exercise factor is a crude approximation of this behavior. In an effort to estimate the fair value of the employee stock option more accurately, one may take the approach that the exercise behavior for a given award will be similar to the exercise behavior for a previous award by the same company. If a company keeps detailed records of the disposition of its employee stock options, a specific technique maybe employed.

This technique may be called "Black-Scholes with Term Structure to Exercise." Rather than selecting a single "average" or "implied" term to exercise, a frequency distribution is constructed from historical data. When normalized by the total number of options issued, each entry in the distribution corresponds to the probability of exercise during the corresponding time period. Once this is available, the fair value vs. time to exercise function is created by using the Black-Scholes-Merton equation at various hypothetical times to expiration. Although a constant volatility and risk-free rate may be used for this part of the calculation, if term structure data for interest rate and/or volatility are available, they may be used. Exhibit 2, entitled ***Variable Interest Rate***, presents series of fair values as functions of time to expiration for different volatilities. The risk-free rate corresponding to each time to expiration is the US Treasury coupon strip rate for the specific tenor matching the expiration time. For each value of the volatility, there is a unique fair value corresponding to a given time to expiration from three to ten years, by quarter. It is also possible to construct the series by month or by any other convenient time interval.

To calculate the estimated fair value given a known frequency distribution of exercise, simply calculate the weighted average of fair values over the range of times to expiration. The weighting factor for each time to expiration will be the probability of exercise for each specific time. This approach is a generalization of the implied time to exercise or the SAB 107 Simplified Method. Exhibit 3, ***Variable Time to Exercise***, presents the resulting fair value for a number of different time-to-exercise distributions. In this example, both the volatility and the risk-free rate are treated as constant. Examples A through L correspond to different assumptions of exercise behavior. All of these examples are characterized by a zero forfeiture rate. Forfeiture may be accounted for in a straightforward manner; however, in such cases the sum of the probabilities of exercise over the entire contractual period would be less than one.

Examples

Here we briefly review the various distributions used to calculate fair values using the Black-Scholes with Term Structure to Exercise method:

A: 100% of options exercised immediately upon vesting at 3.00 years. FV = 10.31. Immediate exercise would be favored by recipients who prefer to convert their award to cash in order to diversify their financial holdings.

B: SAB 107 Simplified Method. FV = 14.29. All options exercised at the midpoint between the vesting date and the expiration date, or at 6.50 years. This is a common simplifying assumption which is used in the absence of other data.

C: All options exercised upon expiration, or at 10.00 years. FV = 16.58. This is the theoretical maximum value for the call option using the given conditions. This case represents "optimal exercise."

D: 50% of options exercised upon vesting, 50% exercised at expiration. FV = 13.45. This case represents a “midpoint” between cases A and C.

E: Uniform rate of exercise over the entire post-vesting period. FV = 14.00. This assumption implies that a recipient is equally likely to exercise at any time after vesting.

F: Uniform rate of exercise following a one-year post-vest holding period. FV = 14.50. The post-vest holding period increases fair value.

G: Linear increase in the probability of exercise with time. FV = 15.09. From a starting point of zero exercise in the first quarter, all options are exercised in the remaining time to expiration with a linear increase in rate.

H: Linear decrease in the probability of exercise over time. FV = 12.91. This is the opposite of case G. The preponderance of exercises at shorter terms causes a decrease in fair value.

I: Term to exercise set to seven years. FV = 14.69. This case is illustrative of the assumption of exercise set to a single value of time.

J: Triangular distribution of probability to exercise. Mode = 6.50. Max = 10.00. Min = 3.00. FV = 14.16. This is one of a class of assumptions of a unimodal distribution of time to exercise around a specific point in time.

K: Normal distribution of probability of exercise. Mean of 6.5, standard deviation of 1.0. FV = 14.33. This is another example of a unimodal distribution around a specific point in time.

L: Beta distribution; alpha =0.75, beta = 0.50. FV = 14.62. The beta distribution is a generalized two-parameter distribution with a finite domain. The distribution may take on a variety of forms. This particular example has a local minimum at approximately 5.5 years.

This exercise shows the wide range of fair values which may be realized by varying the assumptions related to time-based exercise behavior. Of course, all assumptions may not be realistic, depending upon the particular circumstances. If historical data on exercise behavior are available, the modified Black-Scholes method described here provides a convenient vehicle to incorporate these data into the valuation of an employee stock option.

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[1] http://www.fintools.com/resources/white-papers/
[2] https://www.sec.gov/interps/account/sab107.pdf
[3] Financial Accounting Standards Board, Topic 718, as updated January 2010, No. 2010-05
[4] *Financial Accounting Standards Board, Financial Accounting Series No. 263-C, December 2004*