Correlation: Its Role in Portfolio Performance and TSR Payout

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An Important Question

A question often raised by our Total Shareholder Return (TSR) valuation clients concerns the level of time and effort we devote to collecting share price data on all the companies in their peer group and to calculating the ***individual correlation coefficients*** between each and every different pair of stocks in the group. For a large peer group such as the S&P 500 or Russell 2000, this task can consume a significant amount of time and computational resources. For a peer group of ***N*** stocks, there are ***N (N+1)/2*** distinct correlation coefficients. ***For a group of 500 stocks, there are 125,250 separate correlations.*** If we consider the use of weekly data over a 10-year period as a basis for our volatility and correlation estimates, there are over 25,000 data points to collect and audit, plus the calculation of 500 separate volatilities and over 125,000 correlations. Organizing and verifying this mass of data requires precision and care. It should not come as a surprise that the development of a reasonable and reliable valuation is not a task which can be accomplished overnight, or even over a weekend!

In this ***Technical Note***, we examine the role of correlation among equity returns in the behavior of portfolios of stocks, particularly in the overall return of the portfolio and the ranking of the individual returns of the stocks in a given portfolio. This issue is central to the accurate estimation of the potential payout of a relative TSR award under existing market conditions. We will start at the beginning and establish exactly why correlations matter. A number of illustrative examples will be provided to demonstrate the magnitude of the effects of equity return correlations, and why such correlations are ignored only at great peril.

Consider an individual stock with a known price history. By selecting an appropriate look-back period, we can calculate a mean return, $\boldsymbol{\mu}$, and a historical volatility, $\boldsymbol{\sigma}$. By convention, the return and volatility are stated in *percent per annum*. If we assume that the returns from the stock are normally distributed (not necessarily true, but a good starting point) we can assign a level of probability to every possible return, either positive or negative. In this note, we follow the common convention of treating all returns as being continuously compounded, so given two successive closing prices separated by one interval of time, the return is defined as

$$\mathbf{r_t = \ln(S(t)/S(t\text{-}1))}$$

where **S(t)** represents the closing price at time **t** and **S(t-1)** is the closing price at the previous interval, time **t-1**. If we observe the price of this stock over a large number of intervals (trading days, weeks, etc.) we will see that the individual returns may be greater or less than the mean value, but appear to be clustered around it. The volatility, $\boldsymbol{\sigma}$, is indicative of the “tightness” or “looseness” of the clustering of the individual returns around the mean, $\boldsymbol{\mu}$. The greater the volatility (which is defined as the standard deviation of the series of individual returns) the more widely dispersed the individual returns are from their mean.

If we wish to model the progress of the price of the stock through time for predictive purposes (such as the valuation of a TSR or ESO), we can think of each successive price as being determined by an overall growth rate (our observed mean rate of return) plus a random component. This “random-walk” model of asset pricing is fundamental to our understanding of market behavior. Using the definitions above, we may model the progress of the stock price through time as

$$S(T) = S(t)\exp[(\mu - \sigma^2/2)(T\text{-}t) + \varepsilon\sigma(T\text{-}t)^{½}]$$

For simplicity, we have assumed that the stock pays no dividend, and that both the mean return and volatility are scaled to the same units of time used to measure **T** and **t**. Inside the exponential function, the first term represents the constant average growth rate, while the second term is the random element. The random variable **ε** is assumed to be Normally distributed with a mean of zero and a standard deviation of one. This equation represents a stock price which grows at rate in constant proportion to its present value, overlaid with a random fluctuation which may be quite large, but which averages out to zero over a long period of time. The magnitude of the individual random fluctuations of day-to-day or week-to-week prices is scaled by the size of the volatility, **σ**.

This model may be used to simulate the price of any number of stocks over time. If the stocks are ***uncorrelated***, the **ε** distributions for each stock are totally independent of one another. Under actual market conditions, this is not often the case. Movements in the prices or the returns of individual stocks are typically dependent to a greater or lesser extent upon a number of economic factors, such as the price of oil, domestic interest rates, GDP growth rate, growth of the national defense budget, etc. If the performance of two different stocks is dependent upon a number of common economic factors, we would reasonably expect the individual returns of these stocks to be related in some way. For example, an increase in the market price of oil will tend to increase the revenues of all companies which produce petroleum. Rising fuel prices, in turn, will increase cost pressure and lower the earnings of all airlines. For this reason, the successive returns of any pair of stocks are not independent, but rather ***correlated*** to some extent. The magnitude and sign of the correlation between the returns of any pair of stocks can be determined from historical data, and expressed as a ***correlation coefficient***. These coefficients may take on a value from +1.0 to -1.0, where a value of 0.0 indicates that the returns are uncorrelated or independent of one another. For two stocks with a time series of returns $r_i(t)$ and $r_j(t)$, the correlation coefficient over a given interval of time, **0** to **T** is expressed as

$$\rho_{ij} = Cov(r_i, r_j) \;/\; \sigma_i\sigma_j$$

The correlation coefficient for the returns of any stock with itself is 1.00. Thus, for any group of **N** stocks, the correlations may be arranged into an **N x N** square matrix with all entries along the diagonal equal to 1.0. Further, since the order of the two stocks does not affect the correlation coefficient, the matrix is symmetric around the main diagonal. The matrix contains **N** elements equal to 1.0, and **N(N+1)/2** distinct elements below the diagonal. Above the diagonal, the elements are a mirror image of the elements below the diagonal.

Implications of Correlation

What does this mean to us in practical terms? Let us first consider the returns and volatilities of portfolios of correlated stocks. In the simplest case, a portfolio of two correlated stocks, we may express the portfolio return as

$$r_P = \omega_1 r_1 + \omega_2 r_2$$

where ω_1 and ω_2 are the weighting factors of the two stocks. These weighting factors must sum to 1.00. In a long-only portfolio, each weighting factor is ≥0, where in long-short portfolios, some of the weighting factors are negative. This is a simple linear relationship, which may be generalized to any number of assets in the portfolio. The return of the portfolio is simply the weighted average return of the component stocks.

The volatility of the two-stock portfolio involves the volatilities of the two individual stocks, as well as the correlation coefficient between them.

$$\sigma_P = [\ \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\rho_{12}\sigma_1\sigma_2\ ]^{1/2}$$

This relationship, which may be generalized to a portfolio of N stocks, is very important.

The first two terms represent the sum of the squares of the weighting factors times the individual volatilities of the individual stocks. These terms are always positive. It is the third term which demands our attention. ***Since the sign of the correlation coefficient, ρ_{12}, may be either positive or negative, it is theoretically (and practically) possible to reduce the volatility of the overall portfolio by adding stocks with volatilities comparable to the other stocks in the portfolio, if the correlation of the additional stocks with the existing stocks is either weak or negative.*** This remarkable conclusion lies at the heart of Modern Portfolio Theory (MPT).

If stocks tend to move together (positive correlation), overall volatility tends to be magnified. If stocks are negatively or inversely correlated, gains in one position will tend to cancel losses in another, which reduces the overall volatility of the portfolio below the level we would expect if all the stocks moved independently of each other. Here are some simple examples to illustrate this effect.

Example 1

Portfolios of Three Stocks			
	Stock A	***Stock B***	***Stock C***
Stock Price at Time t = 0	***10.00***	***15.00***	***20.00***
Annualized Average Return (Observed)	***3.00%***	***5.00%***	***10.00%***
Annualized Volatility	***20.00%***	***12.00%***	***35.00%***
Weighting Factor ω_i	***0.300***	***0.400***	***0.300***
Case I - No correlation			
Matrix of Correlation Coefficients	***1.000***	***0.000***	***0.000***
Annual Returns of Pairs of Stocks	***0.000***	***1.000***	***0.000***
	0.000	***0.000***	***1.000***
Annualized Average Portfolio Return		***5.90%***	
Portfolio Volatility		***13.01%***	

This example shows a portfolio of three uncorrelated stocks. The individual annualized returns, volatilities, and weighting factors are given. Note that the volatility of the portfolio is only 1.01% greater than the volatility of the “quietest” stock in the group, while a premium of 290 bp has been added to the return of the lowest performing stock by mixing in better performing, yet uncorrelated stocks of comparable or greater volatility. This example illustrates the benefits of diversification.

Example 2

Case II - Postively Correlated Returns				
Matrix of Correlation Coefficients		*1.000*	*0.500*	*0.800*
Annual Returns of Pairs of Stocks		*0.500*	*1.000*	*0.600*
		0.800	*0.600*	*1.000*
Annualized Average Portfolio Return			*5.90%*	
Portfolio Volatility			*18.96%*	

Example 2 uses the same three stocks and weighting factors, but changes the correlation coefficients to represent a situation in which the returns of the stocks are strongly correlated with each other. This may occur if the stocks were all selected from the same industry sector. Note that while the overall portfolio return is unchanged, the volatility of the portfolio has increased by almost six percent. This has significant implications, as we shall see later.

Example 3

Case III - Inversely Correlated Returns				
Matrix of Correlation Coefficients		*1.000*	*(0.300)*	*(0.800)*
Annual Returns of Pairs of Stocks		*(0.300)*	*1.000*	*(0.200)*
		(0.800)	*(0.200)*	*1.000*
Annualized Average Portfolio Return			*5.90%*	
Portfolio Volatility			*5.57%*	

Example 3 uses the same set of stocks and weighting factors, but the correlation coefficients have been adjusted to represent a case in which the stocks are all mutually inversely correlated. Again, the portfolio return is unchanged at 5.90%. The volatility is dramatically reduced to less than half its value in the case of uncorrelated stocks.

The implications of the effect of correlation on portfolio volatility are dramatic. In many cases, we can interpret volatility as the inherent risk of a portfolio or instrument. To illustrate this, Table 1 presents the three component stocks as well as the three example portfolios. The expected annualized return and volatility are given in each case, as well as the probability that the stock or portfolio will show a negative return in any given year. ***There are many investors whose perception of quality includes not only the absolute magnitude of the return, but also its stability over time.*** As we can see, knowledge and intelligent management of equity correlation is a critical element of sound portfolio management.

Table 1

Instrument	*Return*	*Volatility*	*P(r < 0)*
Stock A	*3.00%*	*20.00%*	*44.04%*
Stock B	*5.00%*	*12.00%*	*33.85%*
Stock C	*10.00%*	*35.00%*	*38.75%*
Portfolio I	*5.90%*	*13.01%*	*32.51%*
Portfolio II	*5.90%*	*18.96%*	*37.78%*
Portfolio III	*5.90%*	*5.57%*	*14.47%*

Each individual stock in the portfolio has a better than one-third chance of making a negative return in any given year. ***Portfolio I***, of uncorrelated stocks, exhibits a lower probability of a negative return than any of the three component stocks. The most notable result here is the probability of a negative return from ***Portfolio III***, in which the stocks are strongly negatively correlated. The likelihood of a negative return is dramatically reduced, to less than half of that encountered with any of the component stocks.

The Effect of Correlation on Relative TSR Awards

Since correlation has such a dramatic effect on the performance of equity portfolios there is reason to suspect that correlation will be important in the behavior of the peer group of stocks used to determine the ranking and resulting payout from a relative TSR program. In particular, the correlation of the shares of the subject company with the different shares in the peer group can have a strong effect upon the ranking of the total returns of the various shares at the end of the performance period.

Example 4 is a simple illustration of the impact of correlation upon relative share performance. Two groups of stocks are considered. In the first group, the returns of the stocks are non-correlated. The stocks in the second group have the same individual expected returns and volatilities, but are mutually correlated in an arbitrary fashion. The matrix of correlation coefficients is given. The evolution of the three equity prices through a three-year period is simulated using the equation presented at the top of page 2 and a pseudorandom number selected for each stock. For the non-correlated group, the final price, growth rate, total shareholder return and rank within the group are presented.

The price evolution for the correlated group is simulated in a similar fashion. For correlated stocks, the random components of the respective price movements will not depend upon independent random variables denoted by **ε**, but by three correlated random variables with mean of zero, standard deviation of one, and correlation coefficients identical to the correlation coefficients obtained from analysis of historical price data of the subject stocks. This is accomplished by the calculation of a set of correlated random numbers. The specific details of the calculation are presented in our Working Paper entitled ***"Monte Carlo Simulations for Stock Paths,"*** which may be found on our website.

In the case of no correlation, Stocks A, B, and C are ranked 3, 1, and 2 respectively. When correlation is introduced into the simulation, the same set of random numbers yields a ranking of 1, 2, and 3. Although each set of random numbers will yield a different result, it should be apparent that correlation must be accounted for if one wishes to simulate the behavior of a given group of stocks in a fashion which fairly represents market behavior. An accurate simulation is a requirement for the estimation of a fair value which complies with the requirements of ASC Topic 718.

Example 4

	Three Non-Correlated Stocks			Three Correlated Stocks		
	Stock A	Stock B	Stock C	Stock A	Stock B	Stock C
Stock Price at Time t = 0	10.00	15.00	20.00	10.00	15.00	20.00
Annual Growth Rate (Risk-Neutral)	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
Annualized Volatility	20.00%	12.00%	35.00%	20.00%	12.00%	35.00%
Time T (years)	3.0	3.0	3.0	3.0	3.0	3.0
Expected Value (Mean) Return over T years	3.00%	6.84%	-9.38%	3.00%	6.84%	-9.38%
Standard Deviation of Return over T years	34.64%	20.78%	60.62%	34.64%	20.78%	60.62%
Matrix of Correlation Coefficients	1.000	0.000	0.000	1.000	0.750	0.300
Annual Returns of Pairs of Stocks	0.000	1.000	0.000	0.750	1.000	(0.100)
	0.000	0.000	1.000	0.300	(0.100)	1.000
Cholesky T Matrix	1.000	0.000	0.000	1.000	0.000	0.000
	0.000	1.000	0.000	0.750	0.661	0.000
	0.000	0.000	1.000	0.300	(0.491)	0.818
Random Number	0.25301	2.32182	1.12701	0.25301	2.32182	1.12701
Correlated Random Number	0.25301	2.32182	1.12701	2.33248	2.39888	0.97073
Stock Price at Time T	10.59	25.47	30.01	21.77	25.88	27.30
Growth Rate (Annualized)	1.92%	17.65%	13.52%	25.93%	18.18%	10.37%
Total Shareholder Return	5.93%	69.79%	50.04%	117.71%	72.53%	36.48%
Rank	3	1	2	1	2	3
Portfolio Value at Time t = 0		45.00			45.00	
Portfolio Value at Time T		66.07			74.95	
Portfolio Growth Rate (Annualized)		12.80%			17.00%	

Aside from demonstrating the importance of correlation within the peer group and between the peer group and the subject company, these examples illustrate a number of important concepts related to the selection of the peer group in the design of a relative TSR award program. When designing a plan, it is useful to be aware of the consequences of selection of peer group volatility and correlation within the group and to the subject company. These topics will be examined in greater depth in our continuing series of publications.

Although by no means exhaustive, these examples demonstrate that correlation among equity returns has significant effects upon both portfolio performance and the ranking of a specific company's total shareholder return within a peer group. Since we cannot afford to ignore correlation, the time and effort required to collect data and calculate pairwise equity correlations in a TSR valuation is well justified. In upcoming Technical Notes, we will explore the nature of various approximations of peer group behavior which may be made when a complete set of correlations is not available.

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