VOLATILITY TERM STRUCTURE

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Dumas et al. (1996) report the following:

> "Using a sample of S&P 500 index options during the period June 1988 through December 1993, we evaluate the economic significance of the implied deterministic volatility [DV] function by examining the predictive and hedging performance of the DV option valuation model. We find that its performance is worse than that of an ad hoc Black-Scholes model with variable implied volatilities."

Dumas et al. (1998) report the following conclusion:

> "Claims that the Black and Scholes (1973) valuation formula no longer holds in financial markets are appearing with increasing frequency. When the Black-Scholes formula is used to imply volatilities from reported option prices, the volatility estimates vary systematically across exercise prices and time to expiration. Derman and Kani (1994 a & b), Dupire (1994), and Rubinstein (1994) argue that this systematic behavior is driven by changes in the volatility rate of asset returns. They hypothesize that volatility is a deterministic function of asset price and time, and they provide appropriate binomial or trinomial option valuation procedures to account for this.
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> In this paper, we apply the deterministic volatility option valuation approach to S&P 500 index option prices during the period June 1988 through December 1993. We reach the following conclusions. First, although there is unlimited flexibility in specifying the volatility function and it is always possible to describe exactly the reported structure of option prices, our results indicate that a parsimonious model works best in sample according to the Akaike Information Criterion. Second, when the fitted volatility function is used to value options one week later, the DVF ["deterministic volatility function"] model's prediction errors grow larger as the volatility function specification becomes less parsimonious. In particular, specifications that include a time parameter do worst of all, indicating that the time variable is an important cause of over-fitting at the estimation stage. Third, hedge ratios determined by the Black-Scholes model appear more reliable than those obtained from the DVF option valuation model. In sum, 'simpler is better.' "

Engle and Rosenberg (1998) state:

> "Since pricing options in a generic stochastic volatility environment is not a solved problem, we utilize an approximate option pricing formula for *at-the-money* options that may be applied to a variety of volatility models. The use of a single approximate option pricing formula for different volatility models facilitates the derivation of the option hedge parameters using a consistent methodology.
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> This paper uses the following approximate pricing formula, which will be referred to as the Black-Scholes-plug-in formula or BSP. […] In this approximate pricing formula, expected average volatility […] is 'plugged into' the Black-Scholes formula to obtain the stochastic volatility option price."

Heston and Nandi (2000) review different models including the deterministic volatility models of Derman and Kani (1994b), Dupire (1994) and Rubinstein (1994), and choose as a benchmark the ad hoc Black-Scholes model, thus recognizing its advantages.

Brandt and Wu (2002) conclude:

> "We find that although the implied binomial tree model [Derman and Kani 1994 a & b] prices the American style options better than a standard Cox et al. (1979) tree model with constant volatility, it performs no better than an ad-hoc procedure of smoothing Black-Scholes implied volatilities across strike prices and maturities. The ad-hoc model delivers smaller root mean squared valuation errors than the implied tree model for both put and call options, although a formal statistical test indicates that this difference in root mean squared errors is not statistically significant. The ad-hoc model also outperforms the implied tree model in terms of generating theoretical prices that lie more frequently within the bid/ask spreads. Our results complement and confirm the finding of Dumas et al. [(1998)]"

Berkowitz (2003) states:

> "Our approach also can be used to elucidate two points that do not appear to have been well understood thus far. If the implied volatilities are calculated from European options, the ad hoc approach will fail when applied to American options or exotics. This is important – the ad hoc method remains a leading method used by market makers dealing in exotics such as barrier options. […]
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> However, if the volatility surface is calculated from American options, the ad hoc approach succeeds when applied to American options."

The results presented above justify using the usual Black-Scholes model, binomial and trinomial tree models, and other similar models that accept the stock volatility only as a constant. Whenever the term structure of the volatility is available, an equivalent expected volatility can be computed. This equivalent volatility is in turn plugged into the usual models. This approach is expected to work better for *at-the-money* options.

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