

## RISK NEUTRAL VALUATION

**Sorin R. Straja, Ph.D., FRM**

Montgomery Investment Technology, Inc.

200 Federal Street

Camden, NJ 08103

Phone: (610) 688-8111

[sorin.straja@fintools.com](mailto:sorin.straja@fintools.com)

[www.fintools.com](http://www.fintools.com)

The Black-Scholes differential equation together with the corresponding boundary and final conditions reveals that value of an option does not depend on the expected rate of return ( $\mu$ ) of the stock. For example, for a call option, the BS equation and the corresponding conditions are:

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$
$$\begin{array}{ll} t = T & f = \max(S-k, 0) \\ S \rightarrow \infty & f = S-k \\ S \rightarrow 0 & f = 0 \end{array}$$

The equation (including the boundary and final conditions) as such does not involve any parameters that account for the risk preferences of investors. The parameters that do appear in the BS equation (including the boundary and final conditions) are: current stock price; strike price; stock price volatility; and the risk-free rate of interest. All of these parameters are independent of risk preferences.

These observations led financial economists Cox and Ross to develop an important tool known as risk-neutral valuation method of security valuation. The underlying principle states that when pricing options it is valid to assume that the world is risk neutral (where all individuals are indifferent to risk). The resulting option prices are correct not only in a risk-neutral world, but also in the real world. Cox and Ross give the reasoning as follows:

If the stocks prices follow a random motion, the option values must equal the values predicted by the Black-Scholes formula in order to avoid arbitrage opportunities. These formulae should be valid regardless of the average investor's view towards risk because the differential equation does not include as a parameter the expected return of the underlying asset (which is the only measure of investors' risk preferences). As long as the given investment world satisfies the basic assumptions of Black Scholes formula, the values given by the formula will hold true.

A world is risk neutral when the expected return on all assets is the risk free rate of interest. Cox and Ross derived the option valuation formula in a risk-neutral investment world. A risk neutral world is characterized as a place where the investors require no risk premium for their investments (i.e., the investors always demand only the risk free rate of interest as the average



expected return on investment). In such an investment environment, the reason investors are neutral towards risk is because on an average there is no risk.

In conclusion, as long as the basic economic assumptions of the Black-Scholes model are satisfied, the prices obtained using the formula hold true (i.e., if we use the Black-Scholes model in a risk neutral world, we obtain the same option values as in the real world).

## REFERENCES

*Hull, John C*, “Options, Futures and Other Derivatives”, 4<sup>th</sup> Edition, pages 205,248,249. Upper Saddle River, NJ: Prentice Hall; 2000.

*Chriss, Neil A*, “Black-Scholes and Beyond”, pages 190-192. Chicago, IL: Irwin Professional Publishing; 1997.

