PRICE DISTRIBUTION CASE STUDY

Sorin R. Straja, Ph.D., FRM
Montgomery Investment Technology, Inc.
200 Federal Street
Camden, NJ 08103
Phone: (610) 688-8111
sorin.straja@fintools.com
www.fintools.com

TESTING STATISTICAL HYPOTHESES

In order to apply different stochastic models like Black-Scholes, it is necessary to check the two basic assumptions:

- The return rates are normally distributed
- The return rates are uncorrelated

We mention that using the Black-Scholes model we get, as a conclusion, the log-normal distribution of the stock price.

AMERICAN AIRLINES CASE STUDY

As an example, we use the historical data from American Airlines. In the Appendix, the data are listed chronologically, on a weekly basis, for the time period 1/2/87 - 9/20/96. For each date we have the corresponding closing stock price. We mention that there are some missing data, most of them due to holidays.

STEP I - USE ALL AVAILABLE DATA (1/2/87 - 9/20/96)

NORMALITY TEST

For the normality test we use the D'Agostino tests. Departures from normality may be caused by skewness, kurtosis, or both.

- When we test for departures from normality due to skewness, the output includes the skewness coefficient (computed using the usual formula and the EXCEL one), the Z statistic and the corresponding p-value. If we reject the normal distribution hypothesis, we have a probability equal to p to make an error. Particularly for our study case, if we reject the normal distribution hypothesis we make an error with probability 2.7 x 10^{-10}. This error is very small, we are of course ready to take such a small risk, and therefore we conclude that the distribution is not normal due to skewness.
- When we test for departures from normality due to kurtosis, the output includes the kurtosis coefficient (computed using the usual formula and the EXCEL one), the Z statistic and the corresponding p-value. If we reject the normal distribution hypothesis, we have a probability equal to p to make an error. Particularly for our study case, if we reject the normal distribution hypothesis we make an error with

probability 2.1 x 10^{-16}. This error is very small, we are of course ready to take such a small risk, and therefore we conclude that the distribution is not normal due to kurtosis.

- When we test for departures from normality due to either skewness or kurtosis, the output includes the chi-square statistic and the corresponding p-value. If we reject the normal distribution hypothesis, we have a probability equal to p to make an error. Particularly for our study case, if we reject the normal distribution hypothesis we make an error with probability 1.9 x 10^{-23}. This error is very small, we are of course ready to take such a small risk, and therefore we conclude that the distribution is not normal due to either skewness or kurtosis.
- Based upon the D'Agostino tests, because we are faced with both skewness and kurtosis, we conclude that Black-Scholes provides just a rough estimate.

Table 1. Normality tests for all data (1/2/87 - 9/20/96)

TypeTest		Func	
	Skewness		
1	Skewness (standard formula)	1	-0.75241
1	Skewness (Excel formula)	2	-0.75471
1	Z statistic	3	-6.20624
1	p-value	4	2.7E-10
	Kurtosis		
2	Kurtosis (standard formula)	1	5.47981
2	Kurtosis (Excel formula)	2	5.54835
2	Z statistic	3	8.13414
2	p-value	4	2.1E-16
	Omnibus		
3	chi-square statistic	3	104.68168
3	p-value	4	1.9E-23

Test: H0: the return rates are normally distributed
Against: H1: the return rates are not normally distributed due to skewness (1), kurtosis (2) or both (3)

If you reject H0, you make an error with probability equal to the p-value.

p-value	**Dagostino_Conclusion:**
0.01	Black-Scholes provides just a rough estimate.

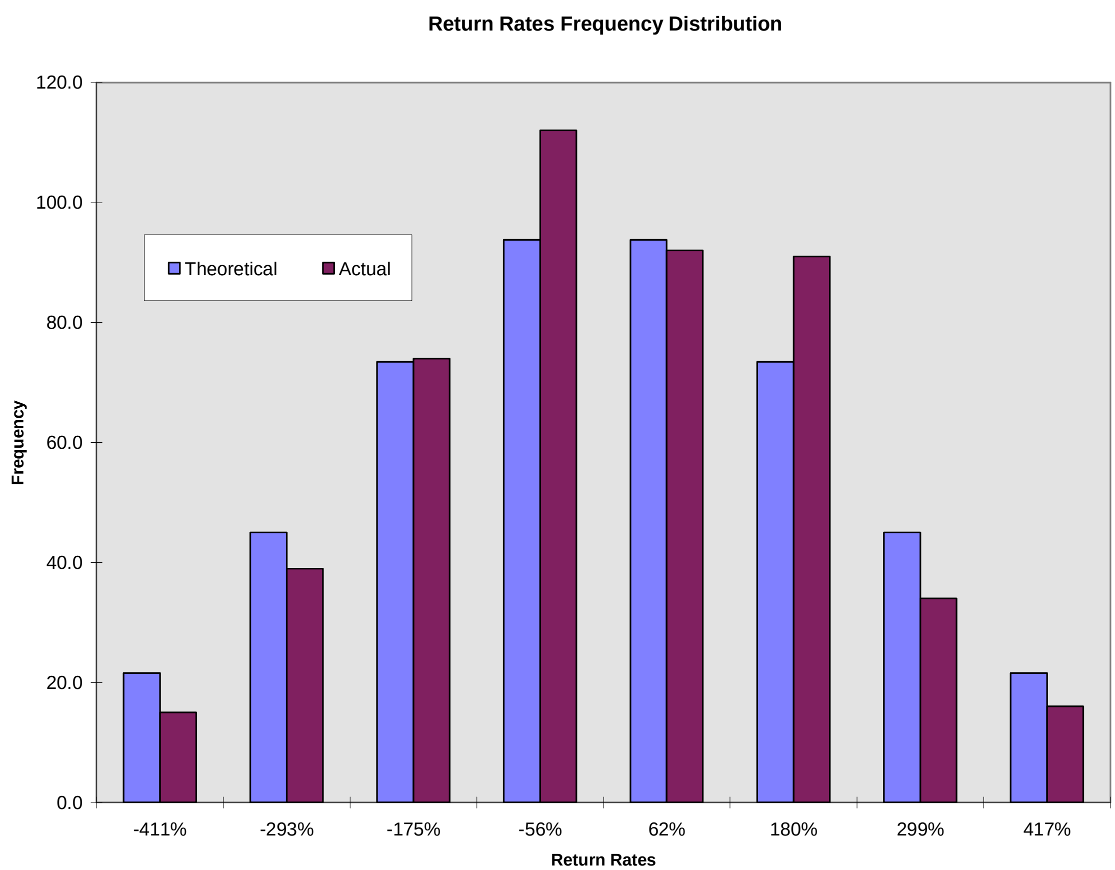
- The above conclusion is provided directly, in plain English, by the FinTools software, whenever we know the risk we are ready to take rejecting the normality hypothesis. Particularly for our study case, for a risk of 1%, the message is "Black-Scholes provides just a rough estimate". For other cases, other potential answers are:
  - Black-Scholes provides a good estimate
  - Black-Scholes overprices out-of-the-money calls and in-the-money puts. It

underprices out-of-the-money puts and in-the-money calls.

- Black-Scholes overprices out-of-the-money puts and in-the-money calls. It underprices in-the-money puts and out-of-the-money calls.
- Black-Scholes underprices out-of-the-money and in-the-money calls and puts.
- Black-Scholes overprices out-of-the-money and in-the-money calls and puts.

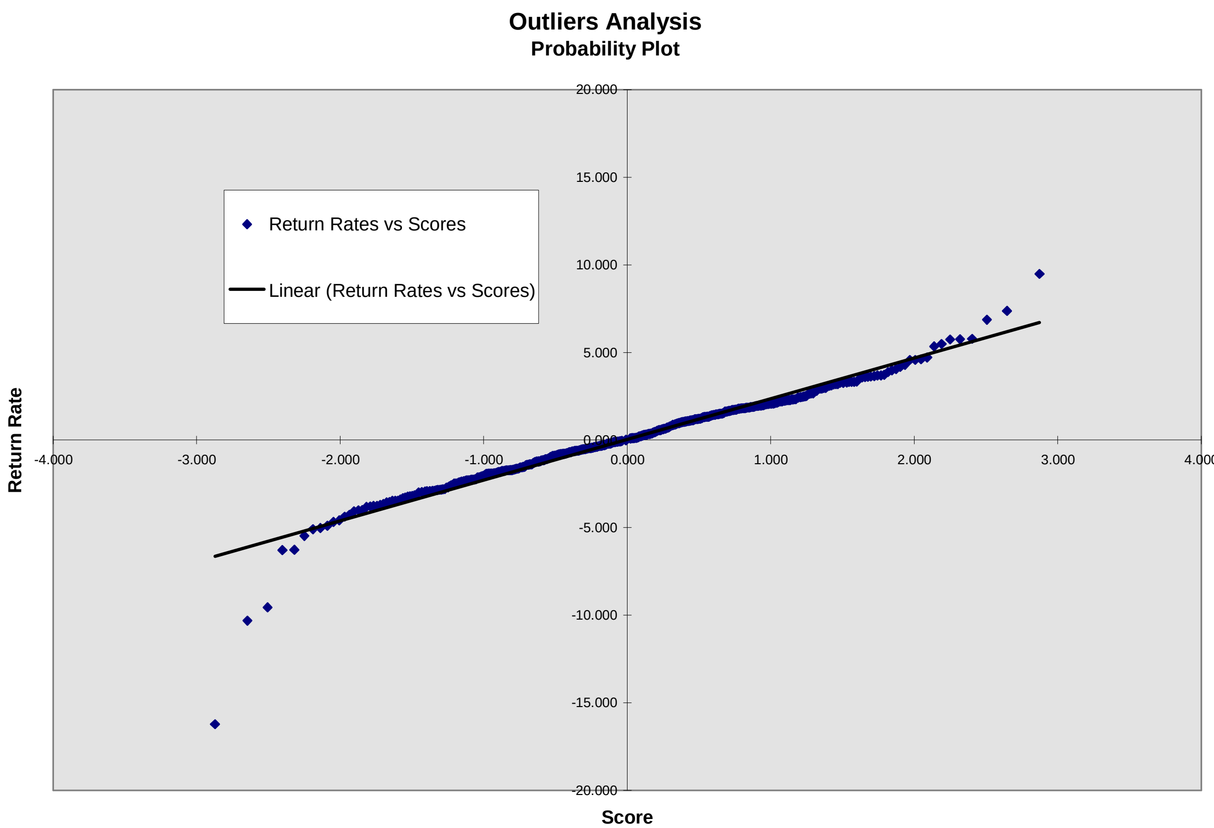
For graphical purposes, we provide a graph with the real histogram and the theoretical normal histogram. We pick up the desired number of classes (always an even number), and as an output we get for each class its mid-point, the real and the theoretical frequencies. Of course we may use the chi-square test to compare the real and theoretical histograms. However, we do not recommend this test, because it is not sensitive enough. The D'Agostino tests presented above are by far more powerful. Particularly for our study case, we can see the significant departure from normality due to both skewness and kurtosis, a fact already diagnosed by the FinTools software.

Figure 1.



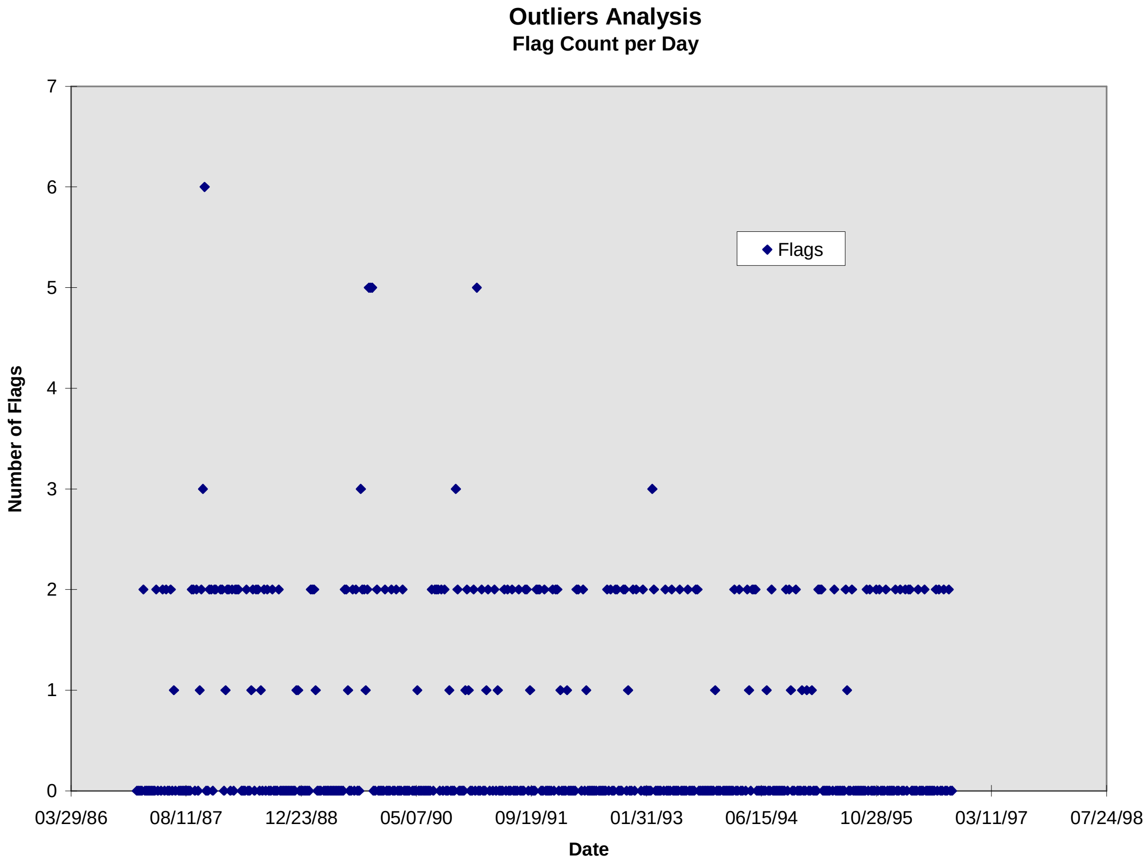
Once we have diagnosed a significant departure from normality, we are interested to know which dates are responsible for this fact. If the distribution is really normal, plotting the return rates as a function of the corresponding scores should result in a diagram where all points lie on a straight line. We can decide what type of scores we want to use (i.e., Blom, Tuckey, or Van der Waerden).

Figure 2. Van der Waerden scores.



Visually inspecting the "return rates - scores" diagram we can identify the outliers and the high leverage points. In order to do this on a statistical basis, we may use the output provided by the FinTools software: it includes the leverage, the standardized residual, the Jacknife residual, the Cook distance, the Welsch & Kuh distance, and the Belsley, Welsch & Kuh distance. One choice is to get the numerical values of the above listed statistics: in this case the user has to identify for each date the correct diagnostic. Another choice is to get directly the diagnostic, instead of the numerical values of these statistics: whenever we are faced with a normal point, the output is zero, while the abnormal points are flagged by an output equal to one. Although the first choice is by far more informative, the second choice may be more useful for the user. In order to easily make a decision, the data may be sorted either chronologically or by scores.

Figure 3.

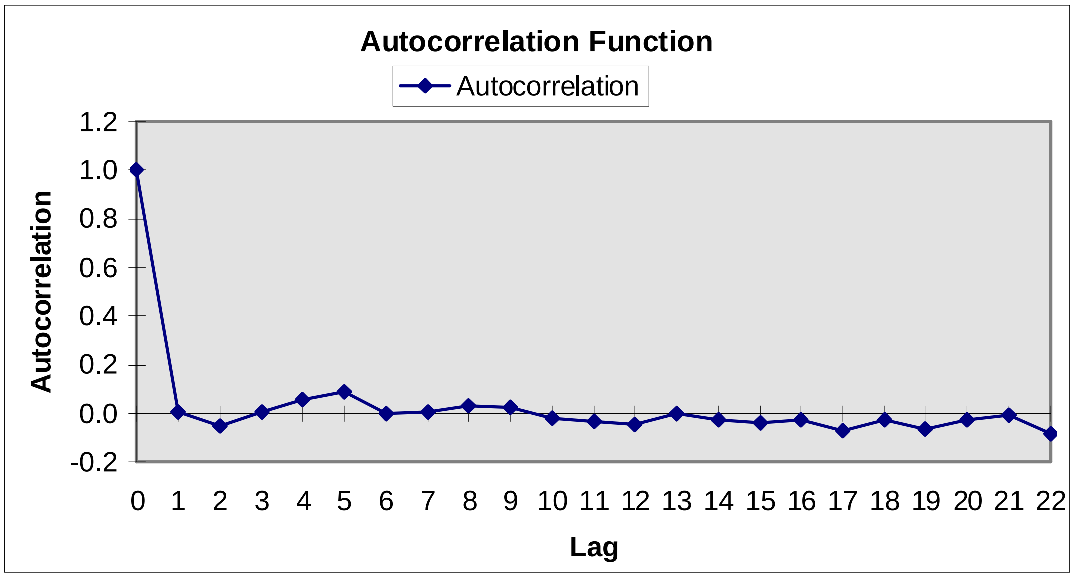


Particularly for our study case, the return rates computed on a weekly basis jump from +948% to -1622% (per annum). The dates with huge absolute value return rates are flagged by most tests. It seems that the middle period exhibits a volatility significantly higher than the beginning or ending period. Using only the statistical tools is not possible to explain why we are faced with this behavior. A direct analysis of the history of the company or the industry may provide the answer. However, for computations affecting future decisions, we should not use all the available data. We have to acknowledge that significant changes took place, and therefore the company we are dealing with in 1996 is significantly different with respect to the company we dealt with in 1987. Based upon this conclusion, we decide to discard all data prior to 5/22/92. We have to repeat our statistical tests using only the data from 5/22/92 until 9/20/96.

CORRELATION TEST

To decide whether or not the return rates are correlated, when the dates are evenly spaced, we may use the autocorrelation function. When the return rates are uncorrelated, the autocorrelation function should be zero for all values of the lag-time excepting the zero lag-time case. Of course, under real circumstances we are faced with autocorrelation functions that match more or less this ideal case. In order to take a decision we may visually inspect the shape of the autocorrelation function. In addition to this, the FinTools software provides the maximum lag-time value to be considered, the Q-statistic, and the corresponding p-value. If we reject the hypothesis that the return rates are uncorrelated, we make an error with probability equal to p. Particularly, for our study case, the maximum lag-time to be considered is 22 weeks, the Q-statistic is 20.8, and the corresponding p-value is 0.53. We are not willing to take a risk of 53%, therefore we cannot reject the hypothesis that the return rates are uncorrelated. We have to point out that the autocorrelation function is estimated assuming evenly spaced data. In our case there are some missing dates, therefore the dates are not always evenly spaced, and henceforth the conclusion should be treated with circumspection.

Figure 4. The autocorrelation function



2	Maximum Lag	22
3	Q Statistic	20.776
4	p-value	53.46%

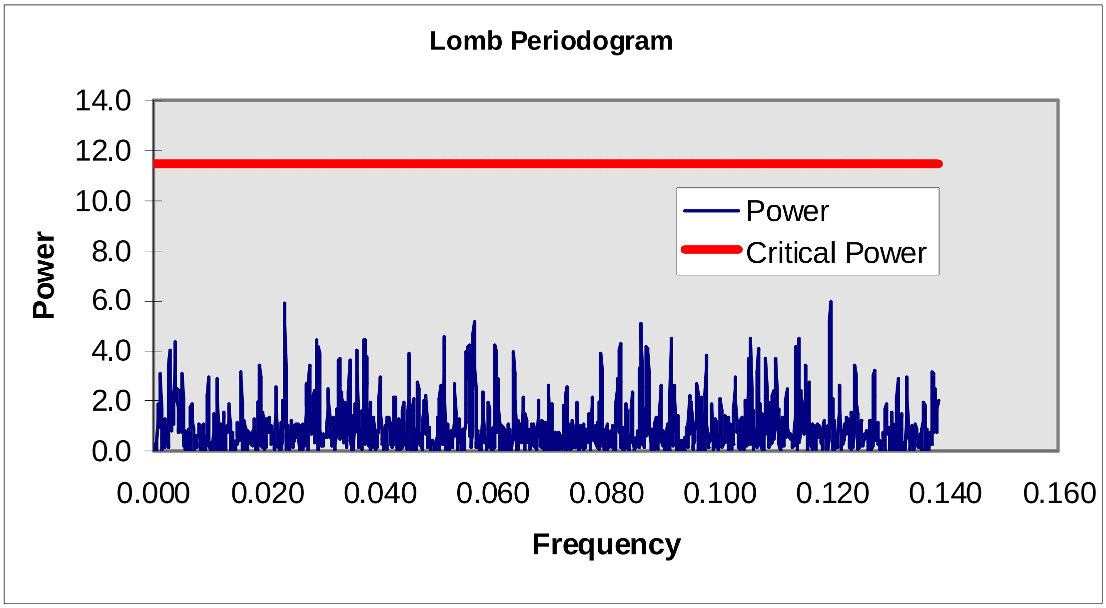
Hypothesis Test:

H0: the return rates are uncorrelated
H1: the return rates are correlated
If you reject H0, you make an error with probability equal to p.

In order to bypass the restriction of evenly spaced data we may use the Lomb periodogram. The FinTools software provides the length of the output arrays, the Lomb periodogram, and the corresponding p-value. If we reject the hypothesis of an uncorrelated noise, we have a probability equal to the p-value to make an error. We have to pick up a significance level, i.e. the risk we are willing to assume when rejecting the non-correlation hypothesis. The Lomb periodogram resembles to a cardiogram: it presents many peaks, some of them may be significant peaks, others may be just background noise. A horizontal straight line corresponds to our significance level: whenever a peak is above this line it is a significant peak, otherwise it is just background noise. If the Lomb periodogram exhibits at least one significant peak, than we should reject the non-correlation hypothesis. Particularly for our study case, the array length is

1964, and the selected significance level is 1%. All peaks are well below the horizontal line corresponding to this significance level, therefore we conclude that the return rates are uncorrelated. The output p-value is 90%, i.e. if we want to reject the non-correlation hypothesis we make an error with probability 90%.

Figure 5. The Lomb periodogram



	Input:	
	Significance Level	1.00%
	Outputs:	
4	p-value=	90.93%
5	Output Array length	1964

If you reject the hypothesis of uncorrelated return rates, you have a probability equal to the p-value to make an error.

STEP II - USE RECENT DATA ONLY (5/22/92 - 9/20/96)

NORMALITY TEST

We basically repeat the same tests using recent dates, only. For the normality test we use the D'Agostino tests.

- If we reject the normal distribution hypothesis due to skewness, we make an error with probability 19.30%. This error is quite high, we are of course not ready to take such a high risk, and therefore we assume that the distribution is normal.
- If we reject the normal distribution hypothesis due to kurtosis, we make an error with probability 37.43%. This error is quite high, we are of course not ready to take such a high risk, and therefore we assume that the distribution is normal.
- If we reject the normal distribution hypothesis due to either skewness or kurtosis, we

make an error with probability 65.25%. This error is quite high, we are of course not ready to take such a high risk, and therefore we assume that the distribution is normal

- Based upon the D'Agostino tests, because we are not faced with either skewness or kurtosis, we conclude that Black-Scholes provides a good estimate.
- The above conclusion is provided directly, in plain English, by the FinTools software, whenever we know the risk we are ready to take rejecting the normality hypothesis. Particularly for our study case, for a risk of 1%, the message is "Black-Scholes provides a good estimate".

Table 2. Normality tests for recent data (5/22/92 - 9/20/96)

TypeTest		Func	
	Skewness		
1	Skewness (standard formula)	1	-0.13967
1	Skewness (Excel formula)	2	-0.14063
1	Z statistic	3	-0.86679
1	p-value	4	0.19303
	Kurtosis		
2	Kurtosis (standard formula)	1	0.02885
2	Kurtosis (Excel formula)	2	0.05743
2	Z statistic	3	0.32046
2	p-value	4	0.37431
	Omnibus		
3	chi-square statistic	3	0.85401
3	p-value	4	0.65246

Test: H0: the return rates are normally distributed
Against: H1: the return rates are not normally distributed
due to skewness (1), kurtosis (2) or both (3)

If you reject H0, you make an error with probability equal to the p-value.

p-value	**Dagostino_Conclusion:**
0.01	Black-Scholes provides a good estimate.

Therefore, for computations affecting future decisions, we may use the data from 5/22/92 until 9/20/96.

CONCLUSION

During the period 1/2/87 - 9/20/96 the company seems to have undergone significant changes. Part of the data should be discarded as past history, and only recent data should be considered as relevant to the today performance of the company.

Based upon statistical tests we assume that the return rates for the period 5/22/92 - 9/20/96 are normally distributed and uncorrelated. These data can be used for computations affecting future decisions.

APPENDIX

Date	Price	Date	Price	Date	Price	Date	Price	Date
1/2/1987	53.75	1/8/1988	35.875	12/23/1988	53.5	12/8/1989	66.125	11/23/199
1/9/1987	55.125	1/15/1988	33.375	12/30/1988	51.75	12/15/1989	64	11/30/199
1/16/1987	56.25	1/22/1988	34.5	1/6/1989	53.25	12/22/1989	60.75	12/7/199
1/23/1987	58.375	1/29/1988	33.125	1/13/1989	53.375	12/29/1989	60	12/14/199
1/30/1987	59.75	2/5/1988	37	1/20/1989	54.375	1/5/1990	58	12/21/199
2/6/1987	56.5	2/12/1988	34.5	1/27/1989	53.25	1/12/1990	59.625	12/28/199
2/13/1987	58.375	2/19/1988	35.375	2/3/1989	58.125	1/19/1990	54.5	1/4/199
2/20/1987	57.875	2/26/1988	39.5	2/10/1989	61.875	1/26/1990	55.75	1/11/199
2/27/1987	58.5	3/4/1988	39.5	2/17/1989	57.875	2/2/1990	57.375	1/18/199
3/6/1987	59	3/11/1988	43.875	2/24/1989	60.375	2/9/1990	53.375	1/25/199
3/13/1987	58.25	3/18/1988	41	3/3/1989	59.625	2/16/1990	54.75	2/1/199
3/20/1987	56.25	3/25/1988	43.125	3/10/1989	58.75	2/23/1990	56	2/8/199
3/27/1987	56.125	4/8/1988	40.875	3/17/1989	61	3/2/1990	57.875	2/15/199
4/3/1987	53.5	4/15/1988	42.25	3/31/1989	58.75	3/9/1990	61.375	2/22/199
4/10/1987	54.5	4/22/1988	41	4/7/1989	59.625	3/16/1990	63	3/1/199
4/24/1987	55.125	4/29/1988	43.375	4/14/1989	59.25	3/23/1990	65.25	3/8/199
5/1/1987	52.125	5/6/1988	43.5	4/21/1989	61.625	3/30/1990	65.5	3/15/199
5/8/1987	52	5/13/1988	44.5	4/28/1989	63.375	4/6/1990	64.5	3/22/199
5/15/1987	55.25	5/20/1988	42.75	5/5/1989	63.375	4/20/1990	63.5	4/5/199
5/22/1987	56	5/27/1988	40	5/12/1989	62.375	4/27/1990	62.75	4/12/199
5/29/1987	54.25	6/3/1988	40.5	5/19/1989	63.5	5/4/1990	61.5	4/19/199
6/5/1987	58.125	6/10/1988	44.875	5/26/1989	64.75	5/11/1990	64	4/26/199
6/12/1987	57.25	6/17/1988	47.5	6/2/1989	64.25	5/18/1990	65.5	5/3/199
6/19/1987	59.875	6/24/1988	46.625	6/9/1989	62.75	5/25/1990	65.75	5/10/199
6/26/1987	59.125	7/1/1988	48.75	6/16/1989	61.375	6/1/1990	63.75	5/17/199
7/10/1987	61.25	7/8/1988	50.375	6/23/1989	61.375	6/8/1990	65.75	5/24/199
7/17/1987	59.125	7/15/1988	47.625	6/30/1989	65.25	6/15/1990	67.125	5/31/199
7/24/1987	60.875	7/22/1988	48	7/7/1989	61.625	6/22/1990	66.125	6/7/199
7/31/1987	61	7/29/1988	45.125	7/14/1989	64.25	6/29/1990	64.375	6/14/199
8/7/1987	62.625	8/5/1988	45.875	7/21/1989	64	7/6/1990	64.125	6/21/199
8/14/1987	65	8/12/1988	45.875	7/28/1989	65.625	7/13/1990	61.125	6/28/199
8/21/1987	64	8/19/1988	42.625	8/4/1989	68.75	7/20/1990	63.5	7/5/199
8/28/1987	63.25	8/26/1988	43.125	8/11/1989	69.5	7/27/1990	59.5	7/12/199
9/4/1987	59.875	9/2/1988	42.75	8/18/1989	74.5	8/3/1990	56.375	7/19/199
9/11/1987	55.375	9/9/1988	43.625	8/25/1989	76.125	8/10/1990	51.625	7/26/199
9/18/1987	57.5	9/16/1988	46.5	9/1/1989	79.125	8/17/1990	51	8/2/199
9/25/1987	55	9/23/1988	45.875	9/8/1989	90.25	8/24/1990	47.5	8/9/199
10/2/1987	55.125	9/30/1988	46.5	9/15/1989	81.25	8/31/1990	46.75	8/16/199
10/9/1987	57.375	10/7/1988	47.5	9/22/1989	75.875	9/7/1990	44.75	8/23/199
10/16/1987	52.75	10/14/1988	49.25	9/29/1989	79.25	9/14/1990	44.5	8/30/199
10/23/1987	46.75	10/21/1988	48.5	10/6/1989	86.5	9/21/1990	43.125	9/6/199
10/30/1987	34.25	10/28/1988	50.25	10/13/1989	103.75	9/28/1990	41.5	9/13/199
11/6/1987	35.25	11/4/1988	48.625	10/20/1989	85.125	10/5/1990	42.625	9/20/199
11/13/1987	35.25	11/11/1988	48.625	10/27/1989	70.875	10/12/1990	42.875	9/27/199
11/20/1987	33.75	11/18/1988	48.125	11/3/1989	70.75	10/19/1990	43	10/4/199
11/27/1987	31.25	11/25/1988	47.625	11/10/1989	73.5	10/26/1990	48	10/11/199
12/4/1987	30.125	12/2/1988	49.75	11/17/1989	69.5	11/2/1990	45.625	10/18/199
12/11/1987	28.375	12/9/1988	52	11/24/1989	67.25	11/9/1990	47.25	10/25/199
12/18/1987	29.75	12/16/1988	53.875	12/1/1989	66.5	11/16/1990	47.25	11/1/199

Date	Price	Date	Price	Date	Price	Date	Price	Date
11/8/1991	63.75	10/30/1992	62.75	10/29/1993	67.375	10/21/1994	51.125	10/6/1995
11/15/1991	60.375	11/6/1992	62.5	11/5/1993	69.75	10/28/1994	52.875	10/13/1995
11/22/1991	60.25	11/13/1992	65.5	11/12/1993	70.25	11/4/1994	55	10/20/1995
11/29/1991	59.75	11/20/1992	63.25	11/19/1993	71.125	11/11/1994	52.625	10/27/1995
12/6/1991	58.5	11/27/1992	61	11/26/1993	68.25	11/18/1994	50.75	11/3/1995
12/13/1991	59.625	12/4/1992	65	12/3/1993	68	11/25/1994	50.625	11/10/1995
12/20/1991	63.75	12/11/1992	64.5	12/10/1993	66.875	12/2/1994	49.5	11/17/1995
12/27/1991	63.25	12/18/1992	61.75	12/17/1993	68.5	12/9/1994	51.625	11/24/1995
1/3/1992	68.125	1/8/1993	63.75	12/31/1993	67.125	12/16/1994	49.75	12/1/1995
1/10/1992	71.625	1/15/1993	69.625	1/7/1994	67.25	12/23/1994	51	12/8/1995
1/17/1992	69.5	1/22/1993	67.75	1/14/1994	69.625	12/30/1994	53.125	12/15/1995
1/24/1992	72.5	1/29/1993	65.5	1/21/1994	71	1/6/1995	53.375	12/22/1995
1/31/1992	70.25	2/5/1993	63.375	1/28/1994	69.75	1/13/1995	55.25	12/29/1995
2/7/1992	70.75	2/12/1993	65	2/4/1994	70.5	1/20/1995	57.75	1/5/1996
2/14/1992	73.25	2/19/1993	62.875	2/11/1994	69.375	1/27/1995	56.875	1/12/1996
2/21/1992	76.5	2/26/1993	55.75	2/18/1994	65.5	2/3/1995	58.5	1/19/1996
2/28/1992	78.625	3/5/1993	59.375	2/25/1994	65	2/10/1995	58	1/26/1996
3/6/1992	76.125	3/12/1993	59.875	3/4/1994	64.25	2/17/1995	60.875	2/2/1996
3/13/1992	75.625	3/19/1993	60	3/11/1994	61.125	2/24/1995	58.25	2/9/1996
3/20/1992	78	3/26/1993	61.375	3/18/1994	62.25	3/3/1995	61.625	2/16/1996
3/27/1992	78.375	4/2/1993	63.5	3/25/1994	61.625	3/10/1995	61	2/23/1996
4/3/1992	74.25	4/16/1993	63.75	4/8/1994	61.5	3/17/1995	61.25	3/1/1996
4/10/1992	70.5	4/23/1993	68.25	4/15/1994	57.875	3/24/1995	61.5	3/8/1996
4/24/1992	69.75	4/30/1993	68.625	4/22/1994	55.625	3/31/1995	62.75	3/15/1996
5/1/1992	65.875	5/7/1993	67.125	4/29/1994	57.25	4/7/1995	64.5	3/22/1996
5/8/1992	66.25	5/14/1993	67.75	5/6/1994	60.75	4/21/1995	67.875	3/29/1996
5/15/1992	69	5/21/1993	71.5	5/13/1994	57.5	4/28/1995	64.875	4/4/1996
5/22/1992	67.125	5/28/1993	71.875	5/20/1994	53.25	5/5/1995	67.125	4/12/1996
5/29/1992	64.75	6/4/1993	71.125	5/27/1994	55.375	5/12/1995	67	4/19/1996
6/5/1992	63.625	6/11/1993	71	6/3/1994	54.875	5/19/1995	69	4/26/1996
6/12/1992	66	6/18/1993	69.375	6/10/1994	56.625	5/26/1995	66.75	5/3/1996
6/19/1992	63.625	6/25/1993	63	6/17/1994	57.75	6/2/1995	66	5/10/1996
6/26/1992	63.75	7/2/1993	62.125	6/24/1994	59	6/9/1995	67.875	5/17/1996
7/10/1992	62.625	7/9/1993	63.625	7/1/1994	57.25	6/16/1995	71.125	5/24/1996
7/17/1992	63.875	7/16/1993	61.5	7/8/1994	59.75	6/23/1995	74	5/31/1996
7/24/1992	65.5	7/23/1993	61.375	7/15/1994	61	6/30/1995	74.5	6/7/1996
7/31/1992	63.75	7/30/1993	65.875	7/22/1994	62	7/7/1995	74.5	6/14/1996
8/7/1992	65.5	8/6/1993	65.875	7/29/1994	59.25	7/14/1995	79.375	6/21/1996
8/14/1992	61.5	8/13/1993	65.25	8/5/1994	57.375	7/21/1995	77.5	6/28/1996
8/21/1992	60.625	8/20/1993	65.375	8/12/1994	57.75	7/28/1995	75.25	7/5/1996
8/28/1992	56.375	8/27/1993	64.5	8/19/1994	56.75	8/4/1995	75.75	7/12/1996
9/4/1992	55.625	9/3/1993	67.875	8/26/1994	56.5	8/11/1995	73	7/19/1996
9/11/1992	57.75	9/10/1993	64.875	9/2/1994	57.75	8/18/1995	70.875	7/26/1996
9/18/1992	61.375	9/17/1993	64.25	9/9/1994	58.875	8/25/1995	73.5	8/2/1996
9/25/1992	58	9/24/1993	64.625	9/16/1994	57.5	9/1/1995	72.25	8/9/1996
10/2/1992	57.125	10/1/1993	64.25	9/23/1994	56.25	9/8/1995	71.5	8/16/1996
10/9/1992	56.625	10/8/1993	65.75	9/30/1994	53.625	9/15/1995	76.625	8/23/1996
10/16/1992	55.125	10/15/1993	64.125	10/7/1994	51.875	9/22/1995	74	8/30/1996
10/23/1992	59.5	10/22/1993	65	10/14/1994	49.125	9/29/1995	70	9/6/1996
								9/13/1996
								9/20/1996