

PARAMETERS ESTIMATION AND CONFIDENCE INTERVALS FOR PREDICTIONS

Sorin R. Straja, Ph.D., FRM

Montgomery Investment Technology, Inc.

200 Federal Street

Camden, NJ 08103

Phone: (610) 688-8111

sorin.straja@fintools.com

www.fintools.com

The simplest stochastic model assumes that the stock price increases due to a return rate affected by white noise perturbation. Using historical data we have to identify the two parameters, the average return rate and the volatility. The FinTools software uses as an input the stock values for given dates. The dates do not have to be evenly spaced. Because always the time interval between two dates is computed as the difference between two calendar dates (neglecting the difference between business days and holidays) there is no adjustment for business days. The stochastic integrals are computed according to the Stratonovich interpretation. As an output, FinTools provides the maximum likelihood estimates of the average return rate and of the volatility. We mention that the maximum likelihood estimate of the volatility is the same for the Ito and Stratonovich interpretations. The standard deviation of the volatility is also computed. For a given confidence level, we get the upper and lower confidence limits of the average return rate. As additional information, for a given starting date with its corresponding stock value, a prescribed time interval and a given number of dates, we get the model prediction for the stock value together with the lower and upper confidence limits.

As an example, we use the historical data from American Airlines. In the Appendix, the data are listed chronologically, on a weekly basis, for the time period 1/2/87 - 9/20/96. For each date we have the corresponding closing stock price. We mention that there are some missing data, most of them due to holidays.

Table 1. Drift and volatility estimates for all data (1/2/87 - 9/20/96)

Input:		
Confidence Level		68.26%
Output:		
	Func	
Drift	1	4.28%
Drift Lower Limit	2	-6.28%
Drift Upper Limit	3	14.85%
Volatility	4	32.92%
Standard Error	5	1.05%

As a result, we get 4.28% for the average return rate, and 32.92% for the volatility.

However, we do not recommend directly using the data without checking first for the basic assumption: normal distribution of the return rates and non-correlation of the return rates. Using the statistical package, we conclude that during the period 1/2/87 - 9/20/96 the company seems to



have undergone significant changes. Part of the data should be discarded as past history, and only recent data should be considered as relevant to the today performance of the company. Based upon statistical tests we assume that the return rates for the period 5/22/92 - 9/20/96 are normally distributed and uncorrelated. These data can be used for computations affecting future decisions.

Table 2. Drift and volatility estimates for recent data (5/22/92 - 9/20/96)

Input:		
Confidence Level		68.26%
Output:		
	Func	
Drift	1	4.48%
Drift Lower Limit	2	-8.30%
Drift Upper Limit	3	17.25%
Volatility	4	26.54%
Standard Error	5	1.27%

Using these data we get 4.48% for the average return rate, and 26.54% for the volatility. Using the confidence limits for the estimates of the average return rate in these two cases we conclude that there is no significant difference. However, the estimates for volatility are significantly different.

We cannot overemphasize the importance of an accurate and correct estimate of the volatility for computing option values.



APPENDIX

Date	Price	Date	Price	Date	Price	Date	Price	Date
1/2/1987	53.75	1/8/1988	35.875	12/23/1988	53.5	12/8/1989	66.125	11/23/1990
1/9/1987	55.125	1/15/1988	33.375	12/30/1988	51.75	12/15/1989	64	11/30/1990
1/16/1987	56.25	1/22/1988	34.5	1/6/1989	53.25	12/22/1989	60.75	12/7/1990
1/23/1987	58.375	1/29/1988	33.125	1/13/1989	53.375	12/29/1989	60	12/14/1990
1/30/1987	59.75	2/5/1988	37	1/20/1989	54.375	1/5/1990	58	12/21/1990
2/6/1987	56.5	2/12/1988	34.5	1/27/1989	53.25	1/12/1990	59.625	12/28/1990
2/13/1987	58.375	2/19/1988	35.375	2/3/1989	58.125	1/19/1990	54.5	1/4/1991
2/20/1987	57.875	2/26/1988	39.5	2/10/1989	61.875	1/26/1990	55.75	1/11/1991
2/27/1987	58.5	3/4/1988	39.5	2/17/1989	57.875	2/2/1990	57.375	1/18/1991
3/6/1987	59	3/11/1988	43.875	2/24/1989	60.375	2/9/1990	53.375	1/25/1991
3/13/1987	58.25	3/18/1988	41	3/3/1989	59.625	2/16/1990	54.75	2/1/1991
3/20/1987	56.25	3/25/1988	43.125	3/10/1989	58.75	2/23/1990	56	2/8/1991
3/27/1987	56.125	4/8/1988	40.875	3/17/1989	61	3/2/1990	57.875	2/15/1991
4/3/1987	53.5	4/15/1988	42.25	3/31/1989	58.75	3/9/1990	61.375	2/22/1991
4/10/1987	54.5	4/22/1988	41	4/7/1989	59.625	3/16/1990	63	3/1/1991
4/24/1987	55.125	4/29/1988	43.375	4/14/1989	59.25	3/23/1990	65.25	3/8/1991
5/1/1987	52.125	5/6/1988	43.5	4/21/1989	61.625	3/30/1990	65.5	3/15/1991
5/8/1987	52	5/13/1988	44.5	4/28/1989	63.375	4/6/1990	64.5	3/22/1991
5/15/1987	55.25	5/20/1988	42.75	5/5/1989	63.375	4/20/1990	63.5	4/5/1991
5/22/1987	56	5/27/1988	40	5/12/1989	62.375	4/27/1990	62.75	4/12/1991
5/29/1987	54.25	6/3/1988	40.5	5/19/1989	63.5	5/4/1990	61.5	4/19/1991
6/5/1987	58.125	6/10/1988	44.875	5/26/1989	64.75	5/11/1990	64	4/26/1991
6/12/1987	57.25	6/17/1988	47.5	6/2/1989	64.25	5/18/1990	65.5	5/3/1991
6/19/1987	59.875	6/24/1988	46.625	6/9/1989	62.75	5/25/1990	65.75	5/10/1991
6/26/1987	59.125	7/1/1988	48.75	6/16/1989	61.375	6/1/1990	63.75	5/17/1991
7/10/1987	61.25	7/8/1988	50.375	6/23/1989	61.375	6/8/1990	65.75	5/24/1991
7/17/1987	59.125	7/15/1988	47.625	6/30/1989	65.25	6/15/1990	67.125	5/31/1991
7/24/1987	60.875	7/22/1988	48	7/7/1989	61.625	6/22/1990	66.125	6/7/1991
7/31/1987	61	7/29/1988	45.125	7/14/1989	64.25	6/29/1990	64.375	6/14/1991
8/7/1987	62.625	8/5/1988	45.875	7/21/1989	64	7/6/1990	64.125	6/21/1991
8/14/1987	65	8/12/1988	45.875	7/28/1989	65.625	7/13/1990	61.125	6/28/1991
8/21/1987	64	8/19/1988	42.625	8/4/1989	68.75	7/20/1990	63.5	7/5/1991
8/28/1987	63.25	8/26/1988	43.125	8/11/1989	69.5	7/27/1990	59.5	7/12/1991
9/4/1987	59.875	9/2/1988	42.75	8/18/1989	74.5	8/3/1990	56.375	7/19/1991
9/11/1987	55.375	9/9/1988	43.625	8/25/1989	76.125	8/10/1990	51.625	7/26/1991
9/18/1987	57.5	9/16/1988	46.5	9/1/1989	79.125	8/17/1990	51	8/2/1991
9/25/1987	55	9/23/1988	45.875	9/8/1989	90.25	8/24/1990	47.5	8/9/1991
10/2/1987	55.125	9/30/1988	46.5	9/15/1989	81.25	8/31/1990	46.75	8/16/1991
10/9/1987	57.375	10/7/1988	47.5	9/22/1989	75.875	9/7/1990	44.75	8/23/1991
10/16/1987	52.75	10/14/1988	49.25	9/29/1989	79.25	9/14/1990	44.5	8/30/1991
10/23/1987	46.75	10/21/1988	48.5	10/6/1989	86.5	9/21/1990	43.125	9/6/1991
10/30/1987	34.25	10/28/1988	50.25	10/13/1989	103.75	9/28/1990	41.5	9/13/1991
11/6/1987	35.25	11/4/1988	48.625	10/20/1989	85.125	10/5/1990	42.625	9/20/1991
11/13/1987	35.25	11/11/1988	48.625	10/27/1989	70.875	10/12/1990	42.875	9/27/1991
11/20/1987	33.75	11/18/1988	48.125	11/3/1989	70.75	10/19/1990	43	10/4/1991
11/27/1987	31.25	11/25/1988	47.625	11/10/1989	73.5	10/26/1990	48	10/11/1991
12/4/1987	30.125	12/2/1988	49.75	11/17/1989	69.5	11/2/1990	45.625	10/18/1991
12/11/1987	28.375	12/9/1988	52	11/24/1989	67.25	11/9/1990	47.25	10/25/1991
12/18/1987	29.75	12/16/1988	53.875	12/1/1989	66.5	11/16/1990	47.25	11/1/1991



<u>Date</u>	<u>Price</u>	<u>Date</u>	<u>Price</u>	<u>Date</u>	<u>Price</u>	<u>Date</u>	<u>Price</u>	<u>Date</u>
11/8/1991	63.75	10/30/1992	62.75	10/29/1993	67.375	10/21/1994	51.125	10/6/1995
11/15/1991	60.375	11/6/1992	62.5	11/5/1993	69.75	10/28/1994	52.875	10/13/1995
11/22/1991	60.25	11/13/1992	65.5	11/12/1993	70.25	11/4/1994	55	10/20/1995
11/29/1991	59.75	11/20/1992	63.25	11/19/1993	71.125	11/11/1994	52.625	10/27/1995
12/6/1991	58.5	11/27/1992	61	11/26/1993	68.25	11/18/1994	50.75	11/3/1995
12/13/1991	59.625	12/4/1992	65	12/3/1993	68	11/25/1994	50.625	11/10/1995
12/20/1991	63.75	12/11/1992	64.5	12/10/1993	66.875	12/2/1994	49.5	11/17/1995
12/27/1991	63.25	12/18/1992	61.75	12/17/1993	68.5	12/9/1994	51.625	11/24/1995
1/3/1992	68.125	1/8/1993	63.75	12/31/1993	67.125	12/16/1994	49.75	12/1/1995
1/10/1992	71.625	1/15/1993	69.625	1/7/1994	67.25	12/23/1994	51	12/8/1995
1/17/1992	69.5	1/22/1993	67.75	1/14/1994	69.625	12/30/1994	53.125	12/15/1995
1/24/1992	72.5	1/29/1993	65.5	1/21/1994	71	1/6/1995	53.375	12/22/1995
1/31/1992	70.25	2/5/1993	63.375	1/28/1994	69.75	1/13/1995	55.25	12/29/1995
2/7/1992	70.75	2/12/1993	65	2/4/1994	70.5	1/20/1995	57.75	1/5/1996
2/14/1992	73.25	2/19/1993	62.875	2/11/1994	69.375	1/27/1995	56.875	1/12/1996
2/21/1992	76.5	2/26/1993	55.75	2/18/1994	65.5	2/3/1995	58.5	1/19/1996
2/28/1992	78.625	3/5/1993	59.375	2/25/1994	65	2/10/1995	58	1/26/1996
3/6/1992	76.125	3/12/1993	59.875	3/4/1994	64.25	2/17/1995	60.875	2/2/1996
3/13/1992	75.625	3/19/1993	60	3/11/1994	61.125	2/24/1995	58.25	2/9/1996
3/20/1992	78	3/26/1993	61.375	3/18/1994	62.25	3/3/1995	61.625	2/16/1996
3/27/1992	78.375	4/2/1993	63.5	3/25/1994	61.625	3/10/1995	61	2/23/1996
4/3/1992	74.25	4/16/1993	63.75	4/8/1994	61.5	3/17/1995	61.25	3/1/1996
4/10/1992	70.5	4/23/1993	68.25	4/15/1994	57.875	3/24/1995	61.5	3/8/1996
4/24/1992	69.75	4/30/1993	68.625	4/22/1994	55.625	3/31/1995	62.75	3/15/1996
5/1/1992	65.875	5/7/1993	67.125	4/29/1994	57.25	4/7/1995	64.5	3/22/1996
5/8/1992	66.25	5/14/1993	67.75	5/6/1994	60.75	4/21/1995	67.875	3/29/1996
5/15/1992	69	5/21/1993	71.5	5/13/1994	57.5	4/28/1995	64.875	4/4/1996
5/22/1992	67.125	5/28/1993	71.875	5/20/1994	53.25	5/5/1995	67.125	4/12/1996
5/29/1992	64.75	6/4/1993	71.125	5/27/1994	55.375	5/12/1995	67	4/19/1996
6/5/1992	63.625	6/11/1993	71	6/3/1994	54.875	5/19/1995	69	4/26/1996
6/12/1992	66	6/18/1993	69.375	6/10/1994	56.625	5/26/1995	66.75	5/3/1996
6/19/1992	63.625	6/25/1993	63	6/17/1994	57.75	6/2/1995	66	5/10/1996
6/26/1992	63.75	7/2/1993	62.125	6/24/1994	59	6/9/1995	67.875	5/17/1996
7/10/1992	62.625	7/9/1993	63.625	7/1/1994	57.25	6/16/1995	71.125	5/24/1996
7/17/1992	63.875	7/16/1993	61.5	7/8/1994	59.75	6/23/1995	74	5/31/1996
7/24/1992	65.5	7/23/1993	61.375	7/15/1994	61	6/30/1995	74.5	6/7/1996
7/31/1992	63.75	7/30/1993	65.875	7/22/1994	62	7/7/1995	74.5	6/14/1996
8/7/1992	65.5	8/6/1993	65.875	7/29/1994	59.25	7/14/1995	79.375	6/21/1996
8/14/1992	61.5	8/13/1993	65.25	8/5/1994	57.375	7/21/1995	77.5	6/28/1996
8/21/1992	60.625	8/20/1993	65.375	8/12/1994	57.75	7/28/1995	75.25	7/5/1996
8/28/1992	56.375	8/27/1993	64.5	8/19/1994	56.75	8/4/1995	75.75	7/12/1996
9/4/1992	55.625	9/3/1993	67.875	8/26/1994	56.5	8/11/1995	73	7/19/1996
9/11/1992	57.75	9/10/1993	64.875	9/2/1994	57.75	8/18/1995	70.875	7/26/1996
9/18/1992	61.375	9/17/1993	64.25	9/9/1994	58.875	8/25/1995	73.5	8/2/1996
9/25/1992	58	9/24/1993	64.625	9/16/1994	57.5	9/1/1995	72.25	8/9/1996
10/2/1992	57.125	10/1/1993	64.25	9/23/1994	56.25	9/8/1995	71.5	8/16/1996
10/9/1992	56.625	10/8/1993	65.75	9/30/1994	53.625	9/15/1995	76.625	8/23/1996
10/16/1992	55.125	10/15/1993	64.125	10/7/1994	51.875	9/22/1995	74	8/30/1996
10/23/1992	59.5	10/22/1993	65	10/14/1994	49.125	9/29/1995	70	9/6/1996
								9/13/1996
								9/20/1996

