

MEAN-REVERSION JUMP-DIFFUSION

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MODEL DESCRIPTION

The *simplest* mean-reversion jump-diffusion model for spot prices is described by the following equation (Clewlow and Strickland, 2000; Clewlow et al., 2001b):

$$dS = \alpha \cdot (\mu - \Phi \cdot K_m - \ln S) \cdot S \cdot dt + \sigma \cdot S \cdot dz + K \cdot S \cdot dq$$

where:

S = spot price

α = mean reverting intensity

μ = long-term average value of $\ln S$ in the absence of jumps

σ = spot price volatility

dz = Wiener process

K = jump with log-normal distribution: $\ln(1+K) \sim N[\ln(1+K_m) - \gamma^2/2, \gamma^2]$

K_m = mean jump size

γ = standard deviation of the proportional jump (jump volatility)

Φ = average number of jumps per year

dq = Poisson process

This model takes into account the perturbations induced by diffusion ($\sigma \cdot S \cdot dz$) and jumps ($K \cdot S \cdot dq$) with respect to the deterministic trend [$\alpha \cdot (\mu - \Phi \cdot K_m - \ln S) \cdot S \cdot dt$]. The first term on the right-hand-side of this equation describes the path of the spot price as a mean-reverting process; $\Phi \cdot K_m$ is the compensation term required to take into account the jump effect.

The corresponding risk-neutral model is:

$$dS = \alpha \cdot (\mu - \Phi \cdot K_m - \lambda - \ln S) \cdot S \cdot dt + \sigma \cdot S \cdot dz + K \cdot S \cdot dq$$

where λ is the market price of risk.

From the spot prices we have to identify the following six parameters: α , μ , σ , K_m , γ , Φ . If necessary, a seventh parameter, λ , should be identified from the futures prices. The six parameters mentioned above can be identified using the maximum likelihood method (Ball and Torous, 1983; Lien and Strom, 1999; Clewlow and Strickland, 2000) or the moments method (Lien and Strom, 1999; Deng, 1999). Clewlow and Strickland (2000) and Clewlow et al.



(2000b) present an intuitive step-wise method for the parameters identification. An alternative indirect method is presented by Jiang (1998).

Sophisticated mean-reversion jump-diffusion models for spot prices incorporate multiple jump processes, regime-switching, and stochastic volatility (Deng, 1999; Deng, 2000). These models have an analytic solution that requires the numerical integration of complex functions.

Deng et al. (1998) consider two models for the futures prices: the usual geometric Brownian motion, and mean-reversion. They provide analytical solutions for different options on futures contracts: spark spread (or heat rate linked derivatives) and locational derivatives (Deng et al., 1998).

Elliot et al. (2000) present a seasonal mean-reverting model where jumps in the spot price arise from supply shocks due to power plants that come off-line or go on-line in a partially predictable manner. This model is used to value futures and options on futures contracts.

COMMERCIALLY AVAILABLE MODELS

The common models used for the valuation of options on futures contracts are based on the Black (1976) and Schwartz (1997) models (Clewlow and Strickland, 1999b; Clewlow et al., 1999b). ***None*** of these models takes into account the jump effect:

1. The Black (1976) model is a one-factor model that does not include mean-reversion.
2. The Schwartz (1997) one-factor model includes mean-reversion.
3. The Schwartz (1997) two-factor model is a development of the Gibson and Schwartz (1990) model where the first factor is the spot price and the second factor is the instantaneous convenience yield. The process for the instantaneous convenience yield accounts for mean-reversion.
4. For the Schwartz (1997) three-factor model the three factors are the electricity spot price, the instantaneous convenience yield, and the instantaneous interest rate. The processes for both the instantaneous convenience yield and the instantaneous interest rate account for mean-reversion.

For these four models there are analytical solutions for the valuation of futures and options on futures contracts (Clewlow and Strickland, 1999b; Clewlow et al., 1999b; Clewlow et al., 2001c). The key distinguishing feature among these models is the shape of the futures price volatility curve. For short maturity options on long maturity futures contracts the Black (1976) model may be used because the volatility curve is relatively flat for long maturities. However, when the integral of the futures price variance is over a non-flat region of the curve, then one of the Schwartz (1997) models should be used. The one-factor model should be used for short maturity options on short maturity futures. The two-factor model should be used for a large and diversified portfolio of options. The three-factor model does not provide a significant improvement, because the interest rate fluctuations have a minor impact on electricity contracts



(Clewlow and Strickland, 2000).

The inclusion of jumps into these models leads to the loss of the convenient simple analytical solutions (Clewlow and Strickland, 2000).

ALTERNATIVE APPROACH

Most of the work on valuing electricity derivatives focused on stochastic modeling for spot prices. This approach has two significant disadvantages:

1. The convenience yield is not observable;
2. The futures price curve is an output of the model and it may not be consistent with the market observable prices.

For options pricing it may be more convenient to use the futures curve as an input of the models, rather than as an output of the models. Cortazar and Schwartz (1994), Clewlow and Strickland (1999a) and Clewlow et al. (1999a) use the entire futures curve. This approach uses all the information contained in the term structure of the futures prices in addition to the historical volatilities of futures returns for different maturities.

CONCLUSIONS

The classical method to value stock derivatives is based upon the creation of a portfolio including both the underlying stock and the derivate contract. The non-storable nature of electricity presents new challenges to the traditional methods of valuing derivatives. Non-storable electricity may exhibit strong mean-reversion over short time horizons that would be inconsistent with an efficient market for a storable good. The electricity derivatives are usually valued by replicating them with futures contracts, rather than to be valued attempting to store or borrow electricity in the spot market. This approach allows traditional no-arbitrage based methods for derivatives valuation to be applied without the implausible assumption that electricity can be stored. Numerical or analytical solutions are available for different cross-commodity derivatives. In most cases the options are on futures contracts (Clewlow et al., 2000c). However, if the expiration date of the option is the same as the expiration date of the futures contract, because futures prices must converge to spot price, that particular option is in fact an option on the spot electricity price.

The easiest task is to simulate spot electricity prices. The corresponding software requires as an input the following six parameters: α , μ , σ , K_m , γ , Φ . The spot price is usually constant for one hour. Therefore, the time step should not be less than one hour. Due to MS Excel limitations, it may be quite difficult to output an array that is longer than approximately 5,800 cells. If the time step is one hour, it translates into approximately eight months. This is the main reason for the relatively short time periods for which Clewlow and Strickland (2000) simulate their spot price. However, we should have no problem to simulate the spot price for longer time periods as long as we output the basic statistics (average, median, standard deviation, skewness, kurtosis, frequency histogram) only.



The identification of the above mentioned six parameters is a very difficult task. The software alone cannot solve this problem. The intimate knowledge of the process is required. **“Finally, it goes without saying that parameter estimation methods are not a magic wand that can be waved at the data. They must be interpreted in the light of experience and knowledge of the conditions in the market at the time”** (Clewlow et al., 2001a).

The estimation of λ (the market price of risk) once α , μ , σ , K_m , γ , and Φ have been identified should pose no special hurdles when compared with the above listed task.

The valuation of futures and options contracts when closed form analytical solutions are available and all parameters are identified should be relatively easy. However, when numerical solutions must be implemented, we may be faced with a difficult problem.

Before we start developing the software for a specific customer, it is mandatory to understand its objectives:

1. What should be valued? Spot, futures, options on futures, options on spot?
2. What model should be used? Mean-reversion only? Jump-diffusion only? Multiple types of jumps? Seasonal adjustment?
3. Who will identify the required parameters?



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