MONTE CARLO SIMULATIONS FOR THE STOCK PATHS

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We simulate the stock paths using the risk-neutral approach (i.e., all stocks grow at the risk-free interest rate). The returns are simulated using Gaussian (i.e., normally distributed) correlated random numbers. The stochastic model may be viewed as an extension of the usual deterministic model for which the rate of return **(μ)** is viewed as a constant value subjected to noise perturbations. Denoting by the **σ** stock volatility and by **y** the dividend yield, the return of **S** between now (time **t**) and a future time **T** is normally distributed with (Chriss 1997 page 99; Hull 1993 page 212):

- The mean equal to $(\mu - y - \sigma^2/2)(T-t)$; and
- The standard deviation equal to $\sigma\,(T-t)^{1/2}$.

For a single stock, the simulated stock value is:

$$S(T) = S(t)*\exp[(\mu - y - \sigma^2/2)(T-t) + \varepsilon\,\sigma\,(T-t)^{1/2}]$$

In the above equation **ε** is an uncorrelated random number normally distributed with mean equal to zero and standard deviation equal to one.

For **correlated** stocks we must take into account the correlation matrix of the rates of return of those stocks. Using the Cholesky factorization (Jorion 1997 page 242) the correlation matrix **R** is factored as:

$$R = T * T'$$

In the above equation **T** is a lower triangular matrix with zeros on the upper right side, and **T'** is the transpose of **T**. As in the case of uncorrelated stocks, first we generate uncorrelated random numbers normally distributed with mean equal to zero and standard deviation equal to 1.0. These numbers are stored in an array **ε**. The correlated random numbers normally distributed with mean equal to zero and standard deviation equal to 1.0 are computed as:

$$\eta = T * \varepsilon.$$

Using subscript **i** as the stock identifier, the simulated stock values are:

$$S_i(T) = S_i(t)*\exp[(\mu_i - y_i - \sigma_i^2/2)(T-t) + \eta_i\,\sigma_i\,(T-t)^{1/2}]$$

It should be note that under risk-neutral conditions all rates of return are equal to the risk-free interest rate.

REFERENCES

Chriss, N. Black-Scholes and Beyond. Option Pricing Models. Chicago, IL: Irwin; 1997.

Hull, J. C. Options, Futures, and Other Derivative Securities. 2nd edition. Englewood Cliffs, NJ: Prentice Hall; 1993.

Jorion, P. *Value at risk*. Chicago, IL: Irwin; 1997.