IMPLIED VOLATILITY TREES AND THEIR APPLICATION TO EMPLOYEE STOCK OPTIONS

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Usually, the theoretical value of an option computed using the Black-Scholes-Merton (Black and Scholes 1973; Merton 1973) formula is not equal to its market price. According to Black (1992), this discrepancy may be due to one of the following facts:

1. The market price is incorrect
2. The parameters used as input for the theoretical value are incorrect
3. The Black-Scholes-Merton theory is incorrect

The implied volatility tree approach (Derman and Kani 1994; Derman et al. 1996; Chriss1997) takes an extreme view - regarding the discrepancy between the theoretical value of an option and its market value - by assuming that the Black-Scholes-Merton (Black and Scholes 1973; Merton 1973) theory is incorrect while the market is always correct.

To address the discrepancy between the theoretical value of an option and its market value, practitioners build a two-by-two grid with one dimension representing strike prices and another dimension representing expiry. The elements of the grid are the Black-Scholes implied volatilities of the options corresponding to the given strike and expiry. Part of any implementation of the implied volatility tree model is a "function" that takes as input the two-by-two grid mentioned above and interpolates/extrapolates from this grid the volatility for a strike and expiry not present in the array. This interpolation/extrapolation technique alone could be used to estimate the option price for a given strike and expiry. The implied tree is not necessary to estimate only the option price.

Derman and Kani (1994) presented the implied volatility binomial tree, which constitutes the first version of the implied volatility tree. The implied volatility trinomial tree - an improved version of the implied volatility tree - was presented by Derman et al. (1996). Derman et al. (1996) provide two algorithms for the implied volatility trinomial tree for the following cases:

1. The implied volatility is a function of the underlying value, only
2. The implied volatility is a function of the time until expiration, only

In general, the implied volatility is a function of both the underlying value and the time until expiration. Derman et al. (1996) mention that the implied volatility tree can be built up when the implied volatility is a product of some function of the underlying value and another function of the time until expiration (i.e., the volatility is "separable"):

$$\sigma(S, T) = f_1(S) \cdot f_2(T).$$

This requirement regarding the volatility may create problems for the interpolation/extrapolation function. An accurate interpolation/extrapolation function is the main requirement for the implementation of the implied volatility tree.

The implied volatility tree provides a tool (Chriss 1997) to:

1. Calculate the hedge ratios for options. The market itself provides only the option prices
2. Price non-standard and exotic options **together** with their hedge ratios

As mentioned above, to estimate the price of an option that is not listed on the market, it is enough to use the interpolation/extrapolation technique. However, in order to get the option price **together** with its hedge parameter, it may be necessary to use the implied tree. The implied volatility tree appears to be very attractive for active traders in order to be able to hedge their portfolios.

However, the main thrust for employee stock options valuation requirements is not hedging, but accounting for the fair market value. Therefore the implied volatility tree appears to be unnecessarily complicated for the purposes of the Financial Accounting Standards Board (FASB). Using the interpolation/extrapolation technique mentioned above, without building the implied volatility tree, is consistent with the requirements of FASB.

For employee stock options, the time horizon is significantly larger than for frequently traded options. Therefore, the two-by-two grid required by the implied volatility tree is only partially available. This aspect limits the applicability of the implied volatility tree to employee stock options.

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