ABSTRACT

The daily close stock prices are adjusted for dividend distributions because the historical volatility is defined as the standard deviation of the total return on a stock. Dividend distributions, which are part of the total return, affect the historical volatility estimates. As required by FAS 123 Appendix F paragraphs 406 (FASB 1995) the “price change resulting solely from the effect of dividend payment on the stock price must be removed from the price observations used to calculate volatility.” The methodology presented below has been developed following FAS 123 Appendix F paragraphs 406-407 (FASB 1995). Applying this methodology results in a historical volatility that matches the results from FAS 123 Appendix F paragraph 408 (FASB 1995).

FORWARD AND BACKWARD DIVIDEND ADJUSTED STOCK PRICES

We consider a hypothetical portfolio that starts with one share of common stock. On each date when a dividend distribution takes place, the resulting cash is invested immediately in the same stock. The number of shares will increase accordingly:

\[ N_0 = 1 \]  \hspace{1cm}  (eq . 1)
\[ N_i = N_{i-1} \cdot \left(1 + \frac{Div_i}{S_i}\right) \hspace{0.5cm}, \hspace{0.5cm} i = 1, 2, \ldots \]  \hspace{1cm}  (eq . 2)

In the above equation \( N_0 \) is the initial number of shares, \( N_i \) is the number of shares at the end of the \( i^{th} \) date, \( Div_i \) is the amount distributed as dividend on the \( i^{th} \) date, and \( S_i \) is the closing stock value on the \( i^{th} \) date.

The forward-dividend-adjusted stock prices are computed as follows:

\[ Forward S_i = S_i \cdot N_i \hspace{0.5cm}, \hspace{0.5cm} i = 1, 2, \ldots \]  \hspace{1cm}  (eq . 3)
The backward-dividend-adjusted stock prices are computed as follows:

\[
\text{Backward } S_i = S_i \cdot \frac{N_i}{N_{\text{Final}}}, \quad i = 1, 2, \ldots \quad (eq. 4)
\]

In the above equations \(N_{\text{Final}}\) is the number of shares on the final date, \(\text{Forward}_S_i\) is the forward-dividend-adjusted stock price on the \(i^{th}\) date, and \(\text{Backward}_S_i\) is the backward-dividend-adjusted stock price on the \(i^{th}\) date. Equation (3) ensures that the forward-dividend-adjusted stock price reported for the first date is equal to the non-adjusted stock price reported for the same date, while equation (4) ensures that the backward-dividend-adjusted stock price reported for the last date is equal to the non-adjusted stock price reported for the same date.

The forward-dividend-adjusted stock price is the value of the hypothetical portfolio with just one common stock share on the first date assuming immediate reinvestment of the dividends in the same stock. The backward-dividend-adjusted stock price is the value of the hypothetical portfolio that will have exactly one common stock share on the last date assuming immediate reinvestment of the dividends in the same stock. The backward-dividend-adjusted and forward-dividend-adjusted stock prices are different by a multiplicative constant. Both backward-dividend-adjusted and forward-dividend-adjusted stock prices will result in the same the relative return (because \(\frac{\text{Forward}_S_i}{\text{Forward}_S_{i-1}} = \frac{\text{Backward}_S_i}{\text{Backward}_S_{i-1}}\)) and therefore in the same historical volatility. However, the absolute returns are different because \(\text{Forward}_S_i - \text{Forward}_S_{i-1}\) is not always equal to \(\text{Backward}_S_i - \text{Backward}_S_{i-1}\). The forward-dividend-adjusted stock prices provide the correct absolute return if there is just one dividend distribution.

Using the backward-dividend-adjusted or forward-dividend-adjusted stock prices implies an increase in the stock price for the day when the dividend is distributed. It should be noted that FAS 123 Appendix F paragraphs 407 (FASB 1995) states that when “computing historical volatility, dividends must be added to the stock price after the ex-dividend date.” As stated by Henne et al. (2007), the idea behind dividend-adjusted stock prices is to consider a stock price that would have resulted in case of reinvesting the company’s profits, instead of distributing the company’s profits.

The weekly and monthly dividend-adjusted stock prices should be obtained from the daily dividend-adjusted stock prices.

**YAHOO! FINANCE DIVIDEND ADJUSTED STOCK PRICES**

It should be noted that the first two equations listed above can be re-written as follows:

\[
N_{i-1} = N_i \cdot \frac{1}{1 + \frac{\text{Div}_i}{S_i}}, \quad i = 1, 2, \ldots \quad (eq. 5)
\]
\[ N_{\text{Final}} = 1 \]  \hspace{1cm} (eq. 6)

**Yahoo! Finance** (2008) uses the following approach:

\[
F_{i-1} = F_i \cdot \left( 1 - \frac{\text{Div}_i}{S_{i-1}} \right), \quad i = 1, 2, \ldots
\]  \hspace{1cm} (eq. 7)

\[
F_{\text{Final}} = 1
\]  \hspace{1cm} (eq. 8)

\[
\text{AdjS}_i = S_i \cdot F_i, \quad i = 1, 2, \ldots
\]  \hspace{1cm} (eq. 9)

In the above equations, \( F_i \) is the **Yahoo! Finance** factor on the \( i \)th date, and \( \text{AdjS}_i \) is the adjusted stock price on the \( i \)th date.

Using the **Yahoo! Finance** approach is equivalent to a *reduction* in the stock price for the day preceding the dividend distribution, as opposed to an *increase* in the stock price for the day when the dividend is distributed. This approach does not match the results from FAS 123 Appendix F paragraph 408 (FASB 1995) because the relative returns are not accounted for correctly. However, as long as the dividend distributions are relatively small, the error is relatively small.

If there is just one dividend distribution, starting one day before the date when dividends are distributed, using the **Yahoo! Finance** approach ensures that the absolute return, as opposed to the relative return, is accounted for correctly. If there is just one dividend distribution, using the forward-dividend-adjusted stock prices ensures that both the absolute return and relative return are accounted for correctly. However, it should be noted that the backward-dividend-adjusted stock prices provide the correct relative return, but fail to provide the correct absolute return.

**REFERENCES**

