DERIVATIVE VALUATION
FOREIGN-DENOMINATED ASSETS

Sorin R. Straja, Ph.D., FRM
Montgomery Investment Technology, Inc.
200 Federal Street
Camden, NJ 08103
Phone: (610) 688-8111
sorin.straja@fintools.com
www.fintools.com

Usually, an executive option package includes a number of options that will vest according to some relative measure of company performance, such as the Total Shareholders' Return (TSR). The performance measure is usually relative to a peer group of competitors. The domestic (home) market is the market where that company is traded. Let us assume that the group of competitors includes companies traded both in the domestic market and a number of foreign markets. Additionally, let us assume that the relative performance of these companies is expressed in domestic currency terms, i.e. foreign stock prices are adjusted to allow for exchange rate movements.

In order to value such a contract, the returns are simulated using Gaussian (i.e., normally distributed) correlated random numbers. For the stock (**S**) of a single company traded in the domestic market, the stochastic model may be viewed as an extension of the usual deterministic model for which the rate of return **(μ)** is viewed as a constant value subjected to noise perturbations. Denoting by the **σ** stock volatility and by **y** the dividend yield, the return of **S** between now (time **t**) and a future time **T** is normally distributed with (Chriss 1997 page 99; Hull 1993 page 212):

- The mean equal to $(\mu - y - \sigma^2/2)(T-t)$; and
- The standard deviation equal to $\sigma\,(T-t)^{1/2}$.

For a single stock traded in the domestic market, the simulated stock value is:

$$S(T) = S(t)*\exp[(\mu - y - \sigma^2/2)(T-t) + \varepsilon\,\sigma\,(T-t)^{1/2}]$$

In the above equation **ε** is an uncorrelated random number normally distributed with mean equal to zero and standard deviation equal to one.

For **correlated** stock traded in the domestic market we must take into account the correlation matrix of the rates of return of those stocks. Using the Cholesky factorization (Jorion 1997 page 242) the correlation matrix **R** is factored as:

$$R = T * T'$$

In the above equation **T** is a lower triangular matrix with zeros on the upper right side, and **T'** is the transpose of **T**. As in the case of uncorrelated stocks, first we generate uncorrelated random numbers normally distributed with mean equal to zero and standard deviation equal to 1.0. These numbers are stored in an array **ε**. The correlated random numbers normally distributed with mean equal to zero and standard deviation equal to 1.0 are computed as:

$$\boldsymbol{\eta} = \mathbf{T} * \boldsymbol{\varepsilon}$$

We simulate the stock paths using the risk-neutral approach (i.e., all stocks grow at the risk-free interest rate).

For multiple stocks traded in both the domestic and foreign markets, all assets and exchange rates are modeled under the domestic martingale measure (Carrett and Wong 2002; Musiela and Rutkowski 2007).

Suppose that there are $\mathbf{N_d}$ domestic-currency-denominated assets, $\mathbf{N_c}$ exchange rates for the foreign currencies, $\mathbf{N_{f,1}}$ assets denominated in the foreign currency **1**, $\mathbf{N_{f,2}}$ assets denominated in the foreign currency **2**, and so on. We denote by $\mathbf{S_d}$ the assets denominated in the domestic currency, by $\mathbf{S_{fk}}$ the assets denominated in the foreign currency **k**, and by $\mathbf{Q_k}$ the exchange rates for the foreign currencies.

The $\mathbf{N_d}$ domestic-currency-denominated assets are modeled as follows ($\mathbf{i=1, 2, \ldots, N_d}$):

$$S_{d,i}(T) = S_{d,i}(t)*\exp[(r_d - y_i - \sigma_i^2/2)(T-t) + \eta_i\, \sigma_i\, (T-t)^{1/2}]$$

In the above equation $\mathbf{r_d}$ is the domestic risk-free interest rate, $\mathbf{y_i}$ is the dividend yield and $\boldsymbol{\sigma_i}$ is the stock volatility for asset **i**.

The $\mathbf{N_c}$ exchange rates for the foreign currencies, expressed as the price (in the domestic currency) of one unit of foreign currency, are modeled as follows ($\mathbf{k=1, 2, \ldots, N_c}$):

$$Q_k(T) = Q_k(t)*\exp[(r_d - f_k - \sigma_{Q,k}^2/2)(T-t) + \eta_{Nd+k}\, \sigma_{Q,k}\, (T-t)^{1/2}]$$

In the above equation $\mathbf{f_k}$ is the domestic risk-free interest rate for the foreign currency **k**, and $\boldsymbol{\sigma_{Q,k}}$ is the volatility for the exchange rate **k**.

The $\mathbf{N_{f,k}}$ foreign-currency-denominated assets, denominated in the foreign currency **k**, are modeled as follows ($\mathbf{j=1, 2, \ldots, N_{f,k}}$):

$$S_{fk,j}(T) = S_{fk,j}(t)*\exp[(f_k - y_{fk,j} - \sigma_{fk,j}^2/2 - \rho_{kfj}\, \sigma_{fk,j}\, \sigma_{Q,k})(T-t) +$$

$$+ \eta_{Nd+Nc+\Sigma Nf^*+j}\, \sigma_{fk,j}\, (T-t)^{1/2}]$$

In the above formula, $\mathbf{\Sigma Nf^*}$ stands for ($\mathbf{N_{f,1}} + \mathbf{N_{f,2}} + \ldots + \mathbf{N_{f,k-1}}$) if k>1 and zero otherwise, $\mathbf{y_{fk,j}}$ and $\boldsymbol{\sigma_{fk,j}}$ are respectively the dividend yield and the stock volatility for asset **j** denominated in the foreign currency **k**, and is the correlation coefficient for the return rates of the exchange rate **k** and asset **j** denominated in the foreign currency **k**.

Details regarding the mathematical aspects are provided by Ravindran (1998 pages 327-328 and 335-337), Carrett and Wong (2002), and Musiela and Rutkowski (2007 pages 149-172).

In order to address the lack of synchronicity of the different markets it is recommended to use weekly data (rather than daily data) for both the historical volatilities and the correlation matrix.

REFERENCES

Carrett, P.; Wong, B. *Executive Options: Valuation and Projection Methodologies.* Sydney: The Institute of Actuaries of Australia; 2002.

Chriss, N. Black-Scholes and Beyond. Option Pricing Models. Chicago, IL: Irwin; 1997.

Hull, J. C. *Options, Futures, and Other Derivative Securities*. 2nd edition. Englewood Cliffs, NJ: Prentice Hall; 1993.

Jorion, P. *Value at Risk.* Chicago, IL: Irwin; 1997.

Musiela, M.; Rutkowski, M. Martingale *Methods in Financial Modelling*. Berlin: Springer Verlag; 2nd Edition Corrected; 2007.

Ravindran, K. *Customized Derivatives. A step-by-step guide to using exotic options, swaps, and other customized derivatives*. New York, NY: McGraw-Hill; 1998.